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A
RUDIMENTARY TREATISE
ON
MASONRY AND STONECUTTING;

IN WHICH THE
PRINCIPLES OF MASONIC PROJECTION
AND THEIR APPLICATION TO THE CONSTRUCTION OF
CURVED WING WALLS, DOMES, OBLIQUE BRIDGES, AND
ROMAN AND GOTHIC VAULTING

ARE CONCISELY EXPLAINED.

IN THREE SECTIONS:

- SECTION I.—ON THE CONSTRUCTION OF VAULTS AND ARCHES.
„ II.—ON MASONIC PROJECTION.
„ III.—ON PRACTICAL STONECUTTING.

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INTRODUCTION.

I. **THIS** little work has been written in continuation of the articles on Masonry, contained in a previous volume* of the Rudimentary Treatises. The reader will there find an outline of the principles of equilibrium of retaining walls and arches, and a sketch of the operations of the mason, with descriptions of the tools and implements used in stone-cutting. These subjects, therefore, have not been touched upon in the following pages, which are devoted more particularly to the scientific operations of stone-cutting, and to the explanation of the methods by which the mason obtains, from the designs of the architect, the exact shape of each stone in a building, so that when set in its place, it shall exactly fit the adjacent stones, without previous reference to them.

II. The necessity for geometrical projection, in order to construct the moulds and templates by which the mason is guided in his work, must always have existed from a very early period; indeed, it would be impossible to erect a stone building of any architectural pretensions, without first arranging the joints of the masonry on a large drawing, and making full-sized projections of some portions, such as the profiles of the mouldings.

III. It would be interesting to trace the history of descriptive geometry in its application to masonic projection; to examine how far the geometrical rules in current use amongst masons at different periods, arose out of the necessities, so to speak, of the architecture

* Rudiments of the Art of Building—Second Series of Rudimentary Treatises, Vol. I.

of the time when they were practised, and to ascertain what influence they, in their turn, exercised on the character of succeeding styles. Thus, the exquisite profiles of the Greek mouldings are true conic sections, the properties of which were well understood by the Greeks, whilst the corresponding members in Roman buildings are tame and spiritless, and composed of circular curves only. Again, whilst the later works of the Roman age betray a total want of rule and system, the architecture of the middle ages exhibits a very perfect and complex geometrical system of construction, arising naturally out of, and yet quite distinct from, that of the classical architecture of earlier times, and equally removed from that of the Italian revival of classic architecture, which sprung up at the commencement of the 15th century, and which, in the course of the 16th, spread so extensively over Europe, as completely to obliterate, so to speak, all traces of the rules of the mediæval architects.

IV. The history of the geometrical methods practised at different periods is not, however, merely a matter of antiquarian interest, but is also an essential branch of knowledge, in connection with the art of stone-cutting. The character of all genuine architecture, no matter of what age or country, is so dependant on its mechanical structure, that we cannot successfully imitate the style of any period, without thoroughly understanding the principles of construction which prevailed at that time. This is especially the case with Gothic masonry, which cannot be properly executed without a thorough appreciation of the peculiar characteristics of mediæval architecture, and of the essential differences which exist between the methods of the Gothic masons and those of our own day, which are almost exclusively derived from the practice of the Italian school of architecture.

V. In former times, the mason had probably little general acquaintance with the principles of projection.

Having no occasion for any rules, besides those required by the architecture of his own time, he worked by them without departing from the beaten track, except when some startling architectural novelty rendered a modification of them absolutely necessary. But in the present day the case is quite different. We copy the architecture of all nations and all times; we introduce in our designs every variety of curves*; and we execute our works in every conceivable material, from granite to gutta-percha.

VI. In this absence of any settled principles of design or construction, the mason can no longer work from traditional rules, or confine himself to one particular style of architecture, and it becomes necessary for him to master the principles of his art, that he may be able to invent for each problem that may come before him the solution best adapted to the character of the work in hand.

VII. In selecting and arranging the materials for this little volume, the object aimed at throughout has been, therefore, to lay down general principles rather than to multiply examples, and will be found to differ from most works on stone-cutting in the omission of many problems usually inserted, which are simply so many exercises on the cone, the cylinder, and the sphere, and have reference only to the round forms of the Italian school, whilst we have written at some length on the subject of ribbed vaulting, the principles of which have not been explained, except in compara-

* The nature of the curves made use of in architectural design has a very marked influence on the character of the work. The curves used by the Greeks were principally conic sections, which appear to have been unknown to the Romans. In the genuine specimens of the pointed style, circular curves only, or curves made up of circular arcs of different radii, are employed, although the profiles of the diagonal ribs, in some examples of vaulting, present curves very similar to the ellipse, being struck from three centres. In Italian architecture, elliptical curves, formed by the intersection of cylindrical surfaces, are of constant occurrence. The use of spiral curves as lines of construction, and not merely of decoration, is quite modern, and dates from the introduction of the oblique arch.

tively expensive works of a class not usually to be found on the book-shelves of the mason.

VIII. The work is divided into three sections, as follows:—

Section I.—*On the Construction of Vaults and Arches.*

The problems which present the greatest difficulties in masonry are those relating to vaulting, the perfect execution of which, from the knowledge it requires of projection and of the nature of the lines produced by the intersections of curved surfaces, has always been the severest test to which the skill of the mason can be exposed. We have, therefore, in the first section briefly sketched the history of stone-cutting in connection with this class of problems, for the purpose of explaining the essential characteristics of the two great classes of vaults, viz., the rib and pannel vault of mediæval architecture, and the solid vault of jointed masonry, which belong to the Roman and Italian styles. Several pages also have been devoted to the explanation of the principles of skew masonry, and of the different methods of constructing oblique arches, that have been advocated by different writers.

Section II.—*On Projection.*

IX. The drawings of the architect are usually made on a rectangular drawing-board, the horizontal and vertical lines being drawn with a T square. In the working drawings of the mason the largeness of the scale renders it impossible to make use of such aids, and a considerable amount of care and system is required to produce a large drawing which shall be truly correct.

Again, in the designs of the architect minute accuracy is comparatively of minor importance if the drawings are properly figured, as the mason should be guided by the written dimensions, and not by the actual size, of the different parts of the drawing. But the working

drawings of the mason exhibit the actual sizes of the stones, any inaccuracy in the drawings materially affecting the soundness of the work.

We have, therefore, in the second section given a few hints on the management of large drawings, which may be useful to those who have not learnt, by painful experience, the necessity of minute accuracy.

The subjects treated of in this section are arranged as follows:—

Working Drawings.—Materials; instruments; scales; figuring; copying; platform-work.

Linear Drawing.—Straight lines; protraction of angles; measurement of right-angled triangles; problems relating to circular curves; modes of drawing the ellipse.

Principles of Projection.—Surfaces; solids; problems relating to the projection and development of the cone, cylinder, and sphere; spiral lines; intersections of curved surfaces.

Section III.—*On Practical Stone-Cutting.*

X. There is a class of problems connected with railway masonry that has as yet been very little studied by working masons; we refer to those required in working the wing-walls of bridges. The construction of curved wing-walls, and the nature of the twist of the coping beds, have been explained at greater length than the limits of this little work would at first seem to warrant. But our reason for this has been, that the same rules apply, with trifling modifications, to all constructions built in horizontal courses with conical beds (as for example, to take two instances apparently most dissimilar, a hemispherical dome and the spandril solid of a fan vault); and, therefore, the system of lines here laid down may be considered, to use the words of Professor Willis, “as a general formula which includes many particular instances.”*

* Willis “On the Construction of the Vaults of the Middle Ages.” Transactions of the Royal Institute of British Architects, Vol. I. Pt. 2.

The subjects treated of in the third section are as follows :—

Part I.—GENERAL PRINCIPLES OF STONE-CUTTING.

Formation of Surfaces.—Plane, curved, and winding surfaces.

Solid Angles.—Nature of solid angles; problems relative to the trihedral.

Surfaces of Operation.

Part II.—APPLICATION OF PRINCIPLES TO PARTICULAR CONSTRUCTIONS.

Battering Walls on curved Plans.

Domes.

Arches.—Arches on rectangular plans, circular and elliptical; oblique arches.

Groined Vaulting.—Roman vaulting; ribbed vaulting.

XI. In concluding these introductory remarks, it may be necessary to add that the reader is presumed to have a knowledge of plane and solid geometry, as well as of the elements of plane trigonometry.

As not only acquaintance, but familiarity with these subjects is indispensable to the proper understanding of the more difficult problems in stone-cutting, especially those connected with skew masonry, no purpose would have been answered by inserting in this volume a preparatory treatise on geometry, which must have necessarily been too brief to be of any real value; and the introduction of which would have excluded much matter bearing more immediately on the subject of the work.

E. DOBSON.

Sneinton, Notts,
Nov. 1849.

RUDIMENTS

OF THE

ART OF MASONRY.

SECTION I.

ON THE CONSTRUCTION OF VAULTS AND ARCHES.

VAULTING.

1. THE construction of plain cylindrical vaults in which the faces, beds, and joints of all the stones are plane surfaces, either perpendicular to, or radiating from, the axis of the cylinder, presents no particular difficulties, the only lines that have to be made use of being straight lines and circular curves; and accordingly we find that, from the earliest times, the construction of common cylindrical vaults, both in brick and stone, appears to have been well understood, arched vaults being found amongst the ruins of Nineveh*, whilst arches of brick and stone are still remaining at Thebes† and Saqqara, in evidence of the knowledge of the arch possessed by the ancient Egyptians. Whether the Greeks were acquainted with the principle of the arch, is still a disputed

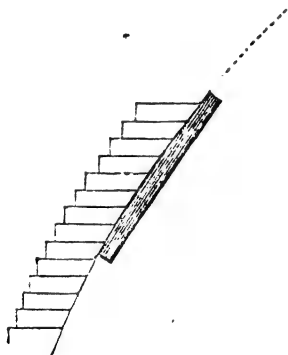
* Vide " Layard's Nineveh."

† Vide " Wilkinson's Manners and Customs of the Ancient Egyptians."

point. Further than this the ancients do not appear to have advanced, and we have no evidence to show that the now familiar problem of finding the profile of a groin from the square sections of a vault by means of ordinates, was at all known before the 11th, or that it was generally practised before the 15th century of the Christian era.

2. The very curious dome-shaped building at Mycenæ, in Greece, known by the name of the "Treasury of Atreus," affords valuable evidence as to the amount of knowledge possessed by its builders of the principles of dome vaulting. The inside of the building forms a pointed dome of 48 ft. diameter, and of about the same height, the section presenting two intersecting arcs of about 70 ft. radius. The difficulties which attend the

Fig. 1.



working of such a vault with radiating beds have been here evaded by making the beds horizontal throughout, the top being formed of a flat stone. Nothing more, therefore, was necessary than to cut the soffit of each course to the required angle with its bed, which could readily be done by means of a templet cut to the radius of the vault, as shown in fig. 1.

3. Although the principle of the arch was known at a very early period, the arch was never employed to any great extent before the Roman age. Its form did not harmonize with the severe horizontal features of the columnar architecture of Egypt and Greece, whilst its employment was not a principle of construction as

amongst the Romans, who built in a great measure with brick, and who probably had not the means of executing the flat massive stone roofs with which the Egyptians covered their halls and porticoes.

4. We must, however, guard against assuming, from the general absence of the arch in Grecian architecture, that the Greek architects were unacquainted with geometrical methods of describing elliptical or any other curves.

The singular facts respecting the curved lines of the Greek temples, which have been recently placed beyond the possibility of dispute by the careful measurements of Mr. F. C. Penrose*, who devoted five months to the investigation of the curves of the Parthenon alone, show that they must have possessed very perfect methods of setting out and executing their work, the perfection of which it would be impossible to excel, and which it would be difficult at the present day to equal. The leading facts to which we refer are briefly these; that the lines of the pavements, architraves, and cornices are not horizontal but curved; and that the entasis or vertical curvature of the columns, and the profiles of the mouldings are true conic sections; being either hyperbolic or parabolic curves. No traces of a knowledge of conic sections are to be found in the architecture of the Romans, whose works are often executed in a coarse and slovenly manner, and whose mouldings are formed of circular curves only, instead of presenting the delicate curves we find in the works of the Greeks.

5. With the introduction of the arch by the Romans as a leading principle of composition, commences a new

* "Two Letters from Athens, by F. C. Penrose, Esq." Published for the Society of Dilettanti.

éra in the history of construction. The arches of Thebes and Ninéveh were of small dimensions and of little importance, but the vaults and domes of the Romans were of such spans as would at the present day, with all our mechanical means and scientific knowledge, be considered bold undertakings. Thus, the dome of the Pantheon, at Rome, is a hemisphere, 139 ft. in diameter, and the groined vaults of the building known by the name of the Temple of Peace were upwards of 70 ft. span. The works of the Romans exhibit great practical knowledge of the equilibrium of arches; and in the building just mentioned, and in the vaulted roofs of the large halls attached to the public baths, we find the arrangement of the groined vault supported by massive arched buttresses, the type of the groined vaults, and flying buttresses of the middle ages.

6. There is, however, no evidence in the works of the Romans of any knowledge of the scientific operations of stone-cutting. Their domes and cupolas could have been constructed with a very simple system of centering, as each course, when completed, became self-supporting, whilst the construction of their groined vaults exhibits an unscientific evasion of constructive difficulties, quite in keeping with the general inattention to minute details, which is one of the characteristic features of Roman work.

7. If two vaults of the same height at the crowns, but of different spans, are to be made to intersect each other, some arrangement is required, in order that the groins, or intersections of the vaulting surfaces, shall lie in vertical planes. In our time, the usual plan adopted is, first to design the curve of the principal vault, and to make the form of the lesser vault dependant upon it, the curve being found from that of the

principal vault by means of ordinates, as shown in fig. 2, plate 1, where the square section of the larger vault is a semicircle, and that of the smaller one a semi-ellipse. This method is, of course, applicable to all cases of intersecting vaults, whatever their curvature may be.

8. This method of finding the profile of a groin by ordinates, from the square section of the principal vault, does not appear to have been known, or at all events practised, by the Romans, and their method of getting over the difficulty was to stilt the springing of the lesser vault, making the sections of both vaults semicircles of different radii. The consequence of this arrangement is, that the vaulting surfaces do not intersect in vertical planes, and the groin forms a waving line, as shown in fig. 3, plate 1. The vaulted roofs of the halls of the Baths of Diocletian and Caracalla are examples of this contrivance, which was also made use of in our own country before the 12th century, when plain cross vaulting began to be superseded by rib and pannel vaulting, which, in its turn, fell into disuse on the revival of the classic style of architecture in the 15th and 16th centuries. In Germany another contrivance appears to have been adopted, which we shall presently describe.

9. So early as the time of Constantine, the art of constructing vaults seems to have been on the decline, and the roofs of the early Christian churches in Italy were of wood, with the exception of the eastern semi-circular apse, which was always covered with a plain semi-dome.

10. In the 6th century was erected the celebrated dome of St. Sophia, at Constantinople. This is a flat dome, 115 ft. in diameter. Soon afterwards was built the church of St. Vitalis, at Ravenna, which has a

hemispherical dome, 54 ft. in diameter. This latter dome is the first example of the re-introduction of dome-vaulting into Italy, after the decline of the Roman art. These two celebrated domes were constructed of earthenware and pumice stone, and presented, consequently, no difficulties in stone-cutting.

After the erection of St. Vitalis, plain groined vaults of small span became very common, although the nave roofs of the Italian churches continued to be constructed of wood, with flat ceilings, until the 13th century, when the pointed style was first introduced into Italy. These vaults are usually divided into compartments, by flat bands, an arrangement which continued to be practised long after the introduction of ribbed vaulting.

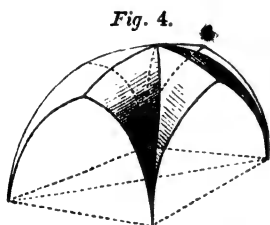
11. The crowns of the Roman vaults were made level throughout, and we find this arrangement to have prevailed in our own country until the introduction of the more complex forms, which we shall presently describe. But, on the Continent a different system seems to have prevailed, the nature of which we shall endeavour to explain.

12. In the construction of a plain waggon vault with cross vaults, the easiest way of forming the centering is to make a complete centering for the main vault, and on it to place the centres for the cross vaults. This dispenses with the necessity for finding the curves of the groins, and the cross vaults may be made of any shape, without regard to their intersection with the main vault, as the groins, to use a familiar phrase, will "*find themselves*." The irregularities of the groin lines of the Roman vaults would seem to indicate that they were built in this way. A centering of this kind is, however, very defective, being weak at the most important parts, namely, under the groins.

The obvious remedy is to construct the centering with diagonal ribs. But here comes the important question—how is the profile of these ribs to be obtained?

13. It is very evident that, for the vaulting surfaces to be cylindrical, the rib must be of a flatter curve than the square section of the vault. If the latter be a semicircle, the former will be a semi-ellipse, and if the form of the vault be pointed, that of the rib will be a pointed arch formed of two elliptical curves. We have already said, that the method of obtaining the profile of a groin by ordinates does not appear to have been formerly known, and in the early German vaults the difficulty is got over in a very simple and satisfactory manner, by abandoning the principle of keeping the

surfaces cylindrical and making the groins portions of circular curves*. The structure of these early vaults is highly domical, the curvature of the groins being such as to throw their intersection much higher than the summit of the transverse



and longitudinal ribs, by which each compartment of the vault was bounded. (See fig. 4.)

14. This expedient does away also with all difficulty arising from the unequal span of two intersecting vaults, and introduced the important principle of designing the profiles of the groins, and leaving the form of the vaulting surface to adapt itself to them, whilst, in the Roman and Italian styles, the form of the vaulting

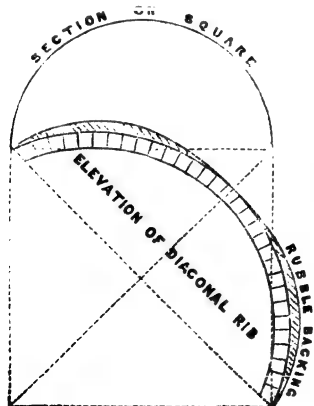
* Probably in many cases a semicircle, to judge from the domed appearance of the vaulting in most of the early German churches; but, in the absence of careful measurements, it is impossible to say what rule was followed in this respect.

surface is first settled, and the profile of the groin follows from it as a matter of necessity. The domical form of vault was extensively used abroad, especially in Italy; but in England it is not common, and our early vaults were constructed on the principle of keeping the crowns level.

15. The early Norman vaults of our own country are plain rubble vaults, similar to those of the Romans, and exhibiting the same expedients of stilted springings and waving groins. But at an early period the system of solid vaults, with continuous vaulting surfaces, began to be superseded by a less massive mode of construction, appropriately called, by Professor Willis, "Rib and pannel work." This style of vault consists of a framework of light stone ribs, filled in with pannels, either built in courses of small stones, or formed of thin slabs, cut to fit the spaces between the ribs.

16. The introduction of diagonal ribs rendered it necessary to make use of some method of obtaining a

Fig. 5.



face-mould for the groins, but this was not done by the methods described above. The common system appears to have been, either to make the diagonal ribs semicircular, and to stilt the springing of the transverse and longitudinal ribs; or, to make the diagonal ribs segmental. In either case, the intersections of the vaulting surfaces rose considerably above the diagonal ribs at the haunches, and, to meet this difficulty,

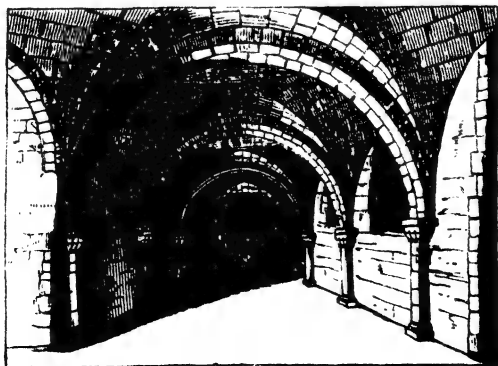
the backs of these ribs were packed up to meet the vault-

ing, which thus rests on thin walls of rubble, instead of on the walls themselves. This is shown in fig. 5. An example of the first-named expedient is to be seen in a vaulted apartment in the castle at Newcastle-upon-Tyne. The aisles of the nave of Peterborough cathedral are examples of the second. Sometimes we find the diagonal ribs semicircular, and the transverse ribs pointed, arches. This construction may be seen in some vaults on the west side of the south transept of Peterborough cathedral.

17. But although the above described arrangements were those in common use, there are instances of plain vaults without diagonal ribs, which present the modern arrangement of making the profile of the groin dependant on the form of the principal vault.

The ruins of some old buildings in Southwark, formerly belonging to the Prior of Lewes, in Sussex, contained vaults of this description. One of them is described in the "*Archæologia*," Vol. XXIII., and also in Brayley's "*Graphic Illustrator*," from which the accompanying illustration, fig. 6, is copied. The length of

Fig. 6.



the vault here shown was 40 ft. 3 in., the width 16 ft. 6 in., and the height 14 ft. 3 in. The main vault was semicylindrical, and was intersected by four cross vaults of elliptical section. The ribs were of stone; the vaultings of chalk. The arch over the entrance doorway of the apartment was also of an elliptical form. The building is supposed to have been erected in the 12th century, but we have no precise information on the subject.

18. It might naturally be expected that the next step in ribbed vaulting, beyond the rude expedient of backing up the diagonal ribs, would have been to accommodate the curvature of the *diagonal ribs* to that of the vaulting surfaces; but, instead of this, we find a new principle of design introduced, which was to adjust the *vaulting surfaces* to the curvature of the ribs, to which they were made perfectly subordinate, each rib being struck from one or more centres, and designed without any immediate reference to the curvature of the adjoining ones.

19. In the Roman system of vaulting, the vaulting surface is everywhere level in a direction parallel to the axis of the vault; and any horizontal section of the spandril of a groined vault taken between the springing and the crown would be a rectangle. But in the Gothic ribbed vault this is not the case, for the plan thus formed would present as many angles as ribs, and admits of great variety according to the curvature of the latter. Thus in fig. 7*, the plan of the spandril at A, by a trifling alteration in the curves of the ribs, might be made at pleasure to form any of the figures shown at a, b, c, and d.

20. The varieties of ribbed vaulting practised during the middle ages may be divided into three classes.

1st. The Plain Ribbed Vault.

2nd. The Lierne Vault; in which numerous *liernes* or short ribs are introduced, disposed in connection with the principal ones, so as to form star-shaped figures round the imposts, as well as a regular pattern at the centre of each compartment.

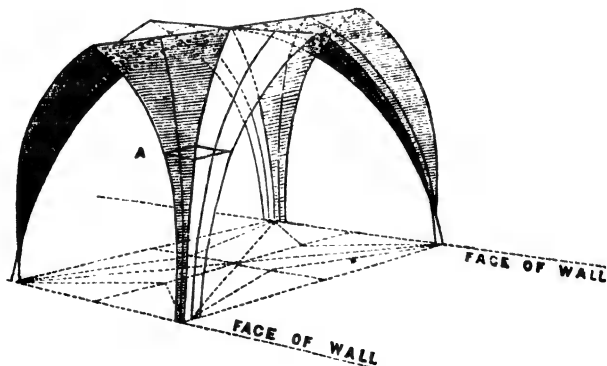
3rd. The Fan Vault; in which all the main ribs have the same curvature, and form equal angles with each other at their springing.

We do not propose to enter upon any description of the architectural design of these vaults or of their decorative features, but it is necessary to say a few words on their mechanical construction.

21. *Plain Ribbed Vaulting*.—A simple example of this is shown in fig. 7. These vaults are sometimes

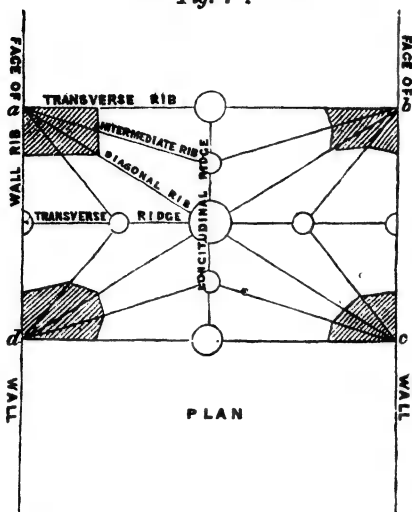
Fig. 7.

PERSPECTIVE VIEW.



found without ridge ribs, and sometimes with them, the latter case being of the most frequent occurrence. Sometimes there are only diagonal, transverse, and

Fig. 7*.



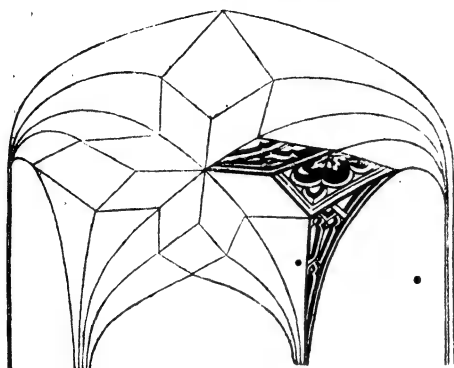
longitudinal ribs; in other examples we find intermediate ribs introduced between the diagonal and transverse, and longitudinal ones. The ridges are generally horizontal, but not universally so.

Plain ribbed vaults were much used in France, and in the Italian churches, and were

often decorated with painting.

22. *Lierne Vaulting*.—In this class of vaults the ribs are very numerous, and the liernes divide the spaces into compartments, which are filled with tracery. In the previous class of vaults, each rib marked a groin; that is, a change in the direction of the vaulting surface; but in these many of the ribs are merely *surface* ribs; that is, they lie in a vaulting surface, whose form is determined independently of them, and regulates their curvature. Many vaults of this class, although apparently of very intricate design, are in reality vaults of simple forms decorated with a profusion of surface ribs. A good example of this kind of vaulting, from the cloisters of St. Stephen's, Westminster, is given in fig. 8. The construction of vaults of this class requires a very thorough knowledge of projection, as the pattern of the vault must be first laid down upon the plan,

Fig. 8.



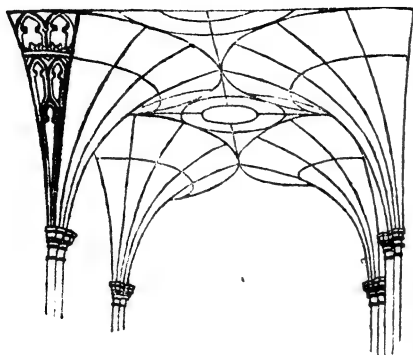
in which the curved lines of the ribs will of course become so foreshortened, that it gives very little idea of the perspective effect of the work in execution. The designers of these vaults must

therefore have possessed the power of conceiving in their minds the effect they wished to produce, and have understood how to distort the plans accordingly.

It is not probable that this was done by any regular geometrical methods; it was more probably the result of experience and observation on the effect of existing vaults. This is confirmed by the very unequal character of remaining examples; in some, the meaning of the design is hardly to be made out from the plans, whilst in others, the plans exhibit symmetrical arrangements, which are lost in execution from the distortion of the lines.

23. Fan Vaulting.—In the fan vault, the main ribs have all the same curvature, and form equal angles with each other: the liernes also are horizontal, each set forming a quadrant, where the vault is divided into rectangular compartments as at King's College Chapel, Cambridge; and where this is not the case, a semicircle, as in the example given in fig. 9, which is from the cloisters of St. Stephen's, Westminster. Lierne and fan vaults were often used in the same building, as in the examples

Fig. 9.



here given from the cloisters of Saint Stephen's, of which the *walks* are covered with fan vaulting, whilst the compartments at the angles are vaulted as shown in fig. 8. But with the invention of the fan vault came also a change in

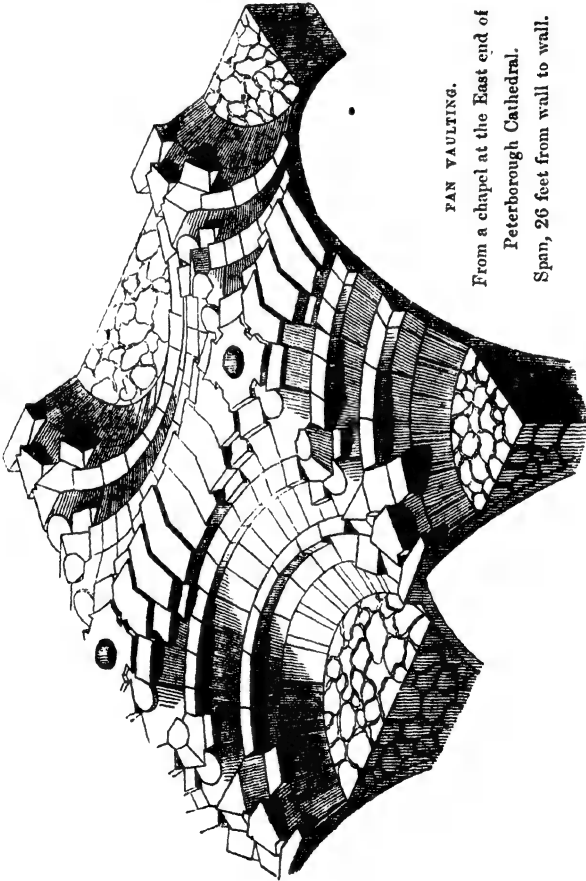
the system of construction, which was also applied to the latter lierne vaults when executed in connection with fan vaults.

24. The early lierne vaults display the same system of construction as the plain ribbed vaults; viz., a skeleton of ribs filled in with thin pannels. In proportion to the complex character of the designs the ribs became more numerous and the pannels smaller, until it was found more convenient to execute the whole vault of jointed masonry, the pannels being sunk in the soffits of the stones instead of being separate stones resting on the ribs. This new system was first introduced in the crowns of the fan vaults, where, from the ramifications of the tracery, the ribs were most crowded, and was soon extended to the construction of the entire vault, although in many instances we find the lower portions, which consist of plain ribs only, to be of ordinary rib and pannel work, whilst the more decorated portions are of jointed masonry. The vaulted roof of King's College Chapel, Cambridge, is an example of the latter mode of construction: that of King Henry the Seventh's

Chapel at Westminster, on the other hand, is built entirely of jointed masonry.

25. The art of stone-cutting appears to have reached its highest development at the commencement of the 16th century; the works of this date exhibiting a per-

Fig. 10.



FAN VAULTING.

From a chapel at the East end of
Peterborough Cathedral.

Span, 26 feet from wall to wall.

fect mastery of the subject. Some idea of the complex character of the masonry of a fan vault may be obtained from an inspection of fig. 10, which is reduced, by permission of Professor Willis, from one of the plates accompanying his valuable paper on the "Construction of the Vaults of the Middle Ages," in the first volume of the Transactions of the Royal Institute of British Architects.

26. Ribbed vaulting was introduced into Italy in the 13th century, the church of St. Andrea di Vercelli, in Piedmont, of which the first stone was laid A.D. 1219, being the first example of its use.

Although the pointed style attained to considerable perfection in Italy, the round forms of the Roman style of vaulting were never entirely superseded. Indeed, the greater part of the Italian ribbed vaults are merely plain vaults with ribs on the groins, and are, in many examples, divided into compartments by the flat band of the earlier round vaultings, which, in the genuine Gothic, became a moulded rib. There are many peculiarities in the Italian ribbed vaults, which mark their distinct character, and show that the pointed style never became perfectly naturalized in Italy. We do not find in them either ridge ribs or liernes, and even the vaulted roof of the cathedral of Milan is a plain ribbed vault, ornamented with *painted* tracery.

27. In Germany and the Netherlands we find lierne vaults of very complex character, some of them exhibiting designs which would seem to have been invented solely for the purpose of showing the skill of the mason in overcoming the difficulty of their execution. The use of fan vaulting appears to have been confined exclusively to our own country.

28. The principal authorities referred to in writing

this brief sketch of the history of ribbed vaulting are—the paper by Professor Willis before referred to; the work by the same author, “Remarks on the Architecture of the Middle Ages, especially of Italy;” “Architectural Notes on German Churches, by the Rev. Dr. Whewell;” and Gally Knight’s “Ecclesiastical Architecture of Italy.” These valuable works cannot be too well studied by those who wish to obtain a clear insight into the principles of ribbed vaulting, as practised in our own and other countries.

29. The abandonment of the principles of the ribbed vault, and the revival of solid vaulting with elliptical groins, may be dated from the commencement of the 15th century. In 1417 Brunelleschi brought forward his plan for the erection of the celebrated cupola over the crossing* of the Duomo at Florence, which was nearly completed at his death, which took place A.D. 1444. This magnificent cupola, which was the first great work of the revival, is built of brick †, like most other Italian domes; it is octagonal in plan, 138 ft. in diameter, and 133 ft. in height, from the springing of the vault to the base of the lantern.

About the same time an Italian architect built the still existing church of the Assumption, at Moscow, of which the vaults are of hewn stone.

30. The Italian architects who flourished during the remainder of the 15th century, followed classic models almost exclusively; and the revival of the columnar styles, and of the round forms of vaulting, gradually spread northwards, although it was not until the middle

* The *crossing* is that part of a cross church at the intersection of the nave and transepts.

† Lined with marbles of different colours.

of the 16th century that the principles of the revival produced any decided effect on the architecture of our own country*.

31. The great master-piece of the modern Italian style of vaulting is the dome of St. Peter's at Rome, 139 ft. in diameter, built at the close of the 16th century, from the designs and instructions left for that purpose by Michel Angelo.

This was only a few years after the completion, in England, of the exquisite vaulted roofs of King's College and Henry the Seventh's Chapel, before alluded to.

The dome of St. Peter's exhibits an advanced knowledge of the application of stone-cutting to domes, being executed of regular masonry; whilst the earlier domes and cupolas were built of bricks, hollow earthenware pots, pumice stone, and similar materials. It is, however, defective in design, from its form not being suited to support the weight of the lantern, and partial failure has taken place.

32. In the year 1568, a century after the erection of the cupola of the Duomo, at Florence, and during the building of St. Peter's at Rome, Philibert De Lorme, a celebrated French architect, published a work on architecture, which contains a complete system of lines for stone-cutting. This is the first published book which treats of masonic projection, all earlier writers being silent on the subject.

In De Lorme's time, ribbed vaulting had fallen into disuse, and he speaks of Gothic vaults, and of the

* The eastern windows of the choir at Lichfield cathedral are filled with stained glass, brought from Germany, the execution of which dates about 1530. The architecture introduced in these paintings is of Italian character, with columns, entablatures, and other features of the revival, which had not then reached England.

methods practised for the adjustment of the curvatures of the ribs, as belonging to a bygone age, considering the works of the Italian school to be the only ones worthy of the name of true architecture. At the same time he acknowledges the extraordinary mechanical skill displayed in their construction, which appears, in his eyes, to have been their chief merit.

33. From De Lorme's time to our own there is little worth noticing in the history of the art. His work was followed by those of other French writers, who copied his constructions, and on the Continent the study of geometrical projection has always formed a prominent branch of the education of the architect.

34. The admirable construction of the vaulted roof* of St. Paul's cathedral attests the knowledge of the architect, and the mechanical skill of the workmen, employed in its construction. But, with the exception of a treatise by Halfpenny, published A.D. 1725, we have no works of that date on stone-cutting, and, indeed, possess scarcely any English publications on the subject, except those published within the last few years, amongst which the works of Mr. Peter Nicholson stand conspicuous for their completeness. Meanwhile, the principles of the construction of the mediæval ribbed vaults seem to have been completely forgotten, and so totally misunderstood, that both Halfpenny and Nicholson give methods, in their works, for constructing Gothic vaults, with diagonal ribs projected from the transverse rib by ordinates; a system which we have shown to be quite at variance with the genuine character of ribbed vaulting.

* The *dome* of St. Paul's is only a wooden covering placed round the brick cone supporting the lantern, and is merely a picturesque addition to the structure, not an essential part of the construction.

OBLIQUE ARCHES.

35. We now come to a new era in the history of the arch. About twenty years ago was introduced a new system of building arches, totally unpractised before in this country; we allude to the erection of oblique or skew bridges built in spiral courses.

Oblique bridges seem to have been known on the Continent long before their introduction into this country, and Vasari mentions one built over the river Mugnone, near Florence, as early as 1530*; but the art does not appear to have been generally understood; for, very recently, the Chevalier Mosca, whilst designing the stone bridge built by him over the Dora Riparia, near Turin, considered the erection of an oblique arch too hazardous an undertaking, and went to a heavy expense in forming new approaches, in order that the bridge should cross the river at right angles to the stream.

36. In England the art of building oblique bridges arose simultaneously with the development of the railway system. Before the introduction of railways, few bridges were built except for carrying common roads over rivers and canals, and such bridges were uniformly erected on a rectangular plan; and, in cases where the direction of the road was not at right angles to the stream to be crossed, the approaches were turned as might be necessary to effect this. The speed of the locomotive engine rendered this arrangement quite inadmissible to bridges erected for carrying railways across

* Vasari. *Vite dei piu eccellenti pittori*. Firenze, 1568. The edition of 1550 contains no notice of this work. The bridge in question was built by Nicolo, surnamed il Tribolo, on the main road to Bologna, outside the gate of San Gallo, at Florence, and seems to have excited much interest at the time of its erection. No details are given of the principles on which it was constructed.

existing communications; and accordingly, with the first introduction of locomotives, arose the necessity for constructing arches on oblique plans.

37. Amongst the first stone skew bridges built in this country of any size was one erected by Mr. John Storey, A.D. 1830, over the River Gaunless, near Durham, on the Hagger Leases Branch Railway, a mineral line joining the Stockton and Darlington Railway.

The angle of this bridge is $26^{\circ} 54'$, the direct span 19 ft., and the oblique span 42 ft., and it was at that time considered a very bold undertaking.

Other skew bridges were built about the same time on the Stockton and Darlington, the Liverpool and Manchester, and other railways; they soon became common, and their construction is now well understood.

38. If an arch could be built in such a manner that the mortar joints should be as strong as the voussoirs themselves, it would signify but little in what direction the courses are built; and the construction of an oblique arch, built either of brick or rubble, offers no difficulty if the cementing material can be depended upon in this respect. But in building with common mortar, or in constructing arches of regular masonry, in which no dependance is placed on the adhesion of the cement, it becomes necessary to place the courses at right angles to the faces of the bridge, in order to bring the thrust of the arch in the right direction, and to keep the obtuse quoins from sliding outwards.

It is this which constitutes the peculiarity of the oblique arch; for the courses not being horizontal, their inclination will be constantly varying, from the springing where it is least, to the crown, where it is greatest, and the accurate working of this *twist*, as it is called, of the

beds, is the great practical problem to be solved in the execution of skew masonry.

39. The ordinary method of building a skew arch, fig. 11, plate 2, is to make it a portion of a hollow cylinder, the arch-stones being laid in parallel spiral courses, and their beds worked in such a manner that in any section of the cylinder perpendicular to its axis, the lines formed by their intersection with the plane of section shall radiate from the axis of the cylinder. In this mode of construction the soffit of each stone will be a portion of a cylindrical surface, and the twist of the beds will be uniform throughout the whole of the arch; so that we have only to settle the amount of the twist, and the stones can then be worked with almost as great facility as the voussoirs of an ordinary arch. The heading joints, or those which divide the stones of each course, are portions of spirals intersecting at right angles the coursing joints, or those which separate the courses, so that the voussoirs are rectangular on the soffit. The quoins, or voussoirs in the faces of the arch, are, however, exceptions to this rule, for the following reason. If a heading spiral be drawn on the centering of an arch, touching the extreme points of the impost, it will lie partly within, and partly beyond, the plane of the face. The heading joints, therefore, will not be parallel to the face-line, and all the quoins will differ more or less from a rectangular form. Another peculiarity of this mode of construction is, that the joints in the face of the arch are not straight, but curved, lines, whose chords will all radiate from a point below the axis of the cylinder, the distance increasing with the obliquity of the bridge.

40. The merit of first explaining the construction of

the oblique arch is due to Mr. Peter Nicholson; who, in 1828, published his "Practical Treatise on Masonry and Stone-cutting," in which directions are given for working the voussoirs of a skew arch in spiral courses. This remained the only work on the subject until 1836, when Mr. Charles Fox published a little pamphlet "On the construction of Skew Arches," which enters into the subject very fully, and explains the mode of working the beds with twisting rules. This was followed in 1839 by Mr. Buck's elegant Treatise, in which the subject is handled with great clearness and simplicity, and trigonometrical formulæ are given for obtaining the dimensions of every part of a skew arch by calculation, instead of by geometrical constructions. In 1845, Mr. Barlow brought out a little pamphlet as a kind of sequel to Mr. Buck's work, containing a diagram for obtaining, by measurement with the scale, most of the data required in the erection of oblique arches. The use of this diagram greatly facilitates the practical application of Mr. Buck's formulæ. In 1839, Mr. Peter Nicholson published his "Treatise on the Oblique Arch," which explains the subject very fully, though not with the conciseness and precision which characterizes Mr. Buck's work. It is, however, a very valuable treatise; and, from the number of problems introduced, is well suited to be put into the hands of the student.

41. All the treatises above mentioned are written with one common object, viz., the construction of cylindrical skew arches in spiral courses, with beds of uniform twist radiating from the axis of the cylinder. It is scarcely necessary to remark that skew arches may be constructed in a variety of ways. Thus an ordinary skew arch, built as above described, is a semicircle, or

some portion of a circle, on the square section, and elliptical on the face, which is an oblique section of a portion of a cylinder. But it is quite possible to make the square section elliptical; in which case the face of the arch will present an elliptical curve, flatter than that of the square section. Again, instead of radiating the bed-joints from the centre of the cylinder, they may be made perpendicular to the curve of the soffit on the oblique section, as in fig. 12, plate 2, which certainly has a better appearance in the elevation of the face of the arch. Both the last-named methods, however, introduce more complexity in the working of the stone; as the twist of the beds will be constantly varying from the springing to the crown, and a great number of twisting rules will be required. So, again, the irregularity in the soffit plans of the face quoins may be done away with by making the heading joints lie in planes parallel to the face of the arch (see fig. 12, plate 2), which gives the soffit a very regular appearance, but weakens the voussoirs by throwing them out of square; the acute angles being liable to be fractured by a very trifling settlement.

In 1837, Mr. John Hart published a "Practical Treatise on the Construction of Oblique Arches," in which these methods are described, with many others which we need not here particularize. The peculiar features of Mr. Hart's system are shown in fig. 12, plate 2, which is taken from the work just mentioned.

42. About the year 1838, Mr. A. I. Adie, then resident engineer on the Bolton and Preston Railway, executed several oblique bridges on that line, the construction of which differs in many respects from the methods above described. The construction of one of these bridges, viz., that over the Lancaster canal, is shown in fig. 13, plate 3, which is copied from the draw-

ings presented by Mr. Adie to the Institution of Civil Engineers, to accompany a paper on these bridges read in session 1839, and is here published by permission of the Institution. The peculiarity in the design of this bridge consists in twisting the coursing joints, so that they shall be perpendicular to all sections of the soffit, made by planes parallel to the face of the arch. The result of this arrangement is, that the courses are not of uniform width, but diverge from the springing, where they are narrowest, to the crown, where they are widest. The square section of the arch is elliptical, not circular, and the bed-joints are worked so as to be everywhere perpendicular to the curve of the soffit on the oblique section.

43. The object proposed by Mr. Adie in the arrangement here described was to bring the thrust of the arch completely parallel to the face, which can only be accomplished approximately with spiral courses of uniform width. But the curved plans of the stones at the springing, and the difficulties which arise in the management of the face joints, from the stones not being of one width, form great obstacles to its general introduction.

44. In the Bath viaduct on the Great Western Railway are two skew arches of peculiar construction. These arches cross the public roads to the west of the Bath Station; they are four centered gothic arches, and are built with courses diverging from the springing to the crown.

45. We have gone to some length in our remarks on the different methods of constructing skew arches in order to induce a careful study of the subject on the part of the reader. In ordinary cases the cylindrical form is the best that can be adopted; but cases may

sometimes occur to which this is inapplicable, and the architect will then find it necessary to adapt the mode of construction to the necessities of the case. No specific rules can be laid down for the treatment of such cases; but the student who has thoroughly mastered the principles of the subject will find no difficulty in applying them in any instance that may occur, however complicated.

SECTION II. ON PROJECTION.

WORKING DRAWINGS.

46. As some of our readers may not be practically acquainted with the routine usually adopted in the erection of large buildings, it may be desirable to say a few words on this subject, that the nature of the working drawings required by the mason may be fully explained.

47. On receiving the designs and instructions of the architect, the mason's first proceeding is to select a convenient spot of ground for a stone-yard as near to the site of the works as practicable, and to erect his workshops and the necessary machinery for lifting the blocks, if the scale of the works be such that they cannot be conveniently moved without mechanical aid.

48. These preliminaries being arranged, the next thing is to order the stone from the quarries that have been chosen; and in order to determine the shapes and sizes of the blocks that will be required, the mason prepares from the designs of the architect a series of drawings

on a large scale, on which he marks the heights of the several courses and the arrangement of the stones in each course, numbering all the stones that require to be worked to definite dimensions. He then makes a schedule of the numbers and sizes of the blocks required, which is sent to the quarry. Each stone being distinguished by its proper number from the time it leaves the quarry to its finally resting in its appointed place in the building, no confusion will arise during the progress of the work; care being taken to number the blocks as nearly as possible in the order in which they are to be set, as attention to this point saves much time and trouble in the execution of the work.

49. Whilst the blocks are being hewn at the quarry, the mason is busily engaged in preparing the rules and templets which will be required for dressing them to their exact shape. For this purpose he lays down on a large wooden floor, or *platform*, full-size plans and sections of the work, course by course, carefully marking the joints according to the working drawings previously made; and from these full-size drawings the templets and bevels are made. Each templet is numbered, to correspond with the number of the block to which it is to be applied, so that no mistake shall occur from working a wrong block, and so wasting the stone. Where the forms of the stones are irregular, a duplicate set of templets is sent to the quarries in order that they may be roughly scappled into shape by the quarryman, which saves expense of carriage, and also much of the subsequent labour of the mason.

50. It will be seen from the above brief outline how much depends upon the accuracy of the working drawings, and how important it is that a mason should be a thorough practical draughtsman. The large size of

many working drawings (as for instance an elevation of a church spire to an inch scale) renders it oftentimes necessary to work on them piecemeal, as it were; and great care and method is required in order to produce a correct drawing. We propose, therefore, to give a few practical hints on the management of large drawings, under the following heads, viz., Materials, Instruments, Scales, Figuring, Copying, and Platform-work.

51. *Materials*.—The best material for working drawing is stout drawing paper mounted on linen, and well seasoned before use. This is somewhat expensive, and for common purposes strong cartridge paper will suffice; but on no account should unmounted paper be used for any but the most temporary purposes, as it is easily torn, and is spoilt by a few hours' exposure in damp weather, whilst drawings on mounted paper will sustain no material injury during many months' rough usage in the workshop and on the scaffold.

52. Indian ink* should be used for the principal lines, red and blue colour being employed for centre lines, and for such lines of construction as it may be desirable to mark in a permanent manner.

Common writing-ink should never be used, nor should any marks be made with it on a drawing, as the first shower of rain to which it may be accidentally exposed causes the ink to run into an unintelligible blot.

53. It is desirable to avoid the use of colour and shading as much as possible, as the use of the brush causes the paper to shrink in those parts where colour has been applied. Indeed, pictorial effect and delicacy

* The ink should be rubbed up fresh every time it is used. Beginners sometimes, to save trouble, content themselves with adding water to ink which has been allowed to dry on the slab. Lines drawn with stale ink are not *fast*, but will smear with the slightest moisture.

of finish are out of place in large working drawings, which should rather be executed with strong lines that will not be effaced by dirt or by the rough handling to which they are exposed; accuracy and neatness are all that is required.

54. *Instruments*.—The principal drawing instruments required by the mason are—the needle-pointer, silk thread, the straight edge and set-square, lead-weights, common and beam compasses, the ruling pen and a set of scales.

55. The *Needle-Pointer* is simply a needle fixed in a short handle, the stump of a pencil for instance. It is used for marking points, which it does in a permanent manner and with greater accuracy than can be obtained by the use of the point of the lead pencil. The pointer is also very useful as a *rest* to keep the straight-edge in its place when drawing long lines; and for copying drawings by pricking through the principal points so as to form corresponding punctures in a sheet of paper placed under the original drawing.

56. *Lead-Weights* are useful for a variety of purposes; but their principal use is to keep the straight-edge steady whilst drawing long lines, or when working a set square against it. Some draughtsmen keep an assistant at their side when setting out the leading lines of large drawings; but it is much more convenient to be quite independent of the assistance of others in these matters, and half a dozen heavy weights and a few pointers will often supply the place of an extra pair of hands.

57. The *Silk Thread* is a reel of strong sewing silk, and is constantly in use for setting out and testing the accuracy of lines which are too long to be drawn with the straight-edge at one operation.

58. The *Straight-Edge* is one of the most important implements used in drawing, as everything depends upon its accuracy. It should be made either of metal or of some tolerably hard wood of uniform texture. Wainscot and mahogany are objectionable materials, but pear-tree and sycamore answer very well. The best way of testing the accuracy of the straight-edge is to compare three together by holding them up against the light, two by two with their working edge in contact. If the light can be seen through them, or if any one of the three do not perfectly coincide with the other two, the edges must be corrected again and again, until this degree of accuracy is obtained.

59. The *Set-Square* requires the same degree of accuracy as the straight-edge; and the straightness of its edges may be tested in the same way. To examine whether the angle contained by the working edges is exactly 90° , draw a straight line on a board, and set up a perpendicular to it by means of the set-square; then reverse the square, and if the edge, when reversed, exactly coincides with the perpendicular just drawn, the square may be considered correct. The lines for a test of this kind should be cut on the board with a drawing knife, as a pencil line is too coarse to be a satisfactory check.

60. Both straight-edges and set-squares should be kept *flat* in a dry place. If hung up against a wall they will warp and soon become untrue.

61. The *Compasses* are used for drawing circular curves. Two pairs are required, one for curves not exceeding 8 in. radius, and another for larger curves up to 15 in. radius.

There are many different constructions of compasses, each of which has some peculiar advan-

tage. The reader may consult on this subject the "Treatise on Mathematical Instruments" of this Series, where he will find engravings and descriptions of all those in common use. These instruments are expensive, but no economy will result from buying inferior ones, which are worse than useless.

62. The curved rulers manufactured in Paris of thin veneer, and sold under the name of *French curves*, are very useful for drawing in between points previously determined small portions of elliptical or other curves, which cannot conveniently be struck from centres.

63. The *Beam-Compass* is used for drawing circular curves from 15 in. to 4 ft. or 5 ft. radius. It is an expensive instrument, but it is indispensable in making drawings on a small scale, in which the curved lines are very close together. See "Treatise on Mathematical Instruments" before referred to. For the purposes of working drawings, however, a very simple and excellent beam-compass may be made, as shown in fig. 14. This

Fig. 14.



instrument consists of a clean pine lath $1\frac{1}{4}$ in. wide, $\frac{1}{8}$ th thick, and about 5 ft. long. At one end is attached a thin piece of veneer with a nick in it, in which to rest the pen or pencil. A slip of drawing paper glued on the upper side of this rule keeps it from splitting, and, being carefully graduated, serves as a scale.

To use this beam-compass a pointer is passed through the lath into the drawing table at the proper distance from the rest, and the pen or pencil is placed in the nick. The only thing to be attended to in the construction of the instrument is to take care that the un-

derside of the rest is raised sufficiently above the underside of the rule, so as not to smear the lines drawn with the pen. The divisions on the scale should be drawn in with curved lines, having the nick for their common centre, by which means the pointer can be set at pleasure in any part of the width of the rule.

For setting out work on a platform, a lath with a bradawl at each end, one as a centre, and the other to mark the curves with, forms a very good beam-compass.

64. *Sweeps*.—When the radius of a curve exceeds 5 ft., it generally becomes necessary to describe it without making use of the centre; and for this purpose *sweeps*, or curved rulers, are used, by means of which the curves are drawn in between points previously ascertained by calculation. These sweeps are made of thin wood, on which the curve is first struck with the trammel as follows:—Find by calculation or otherwise three points in the curve, the middle point being in the centre between the extreme ones or nearly so. Fix a pointer at each of the extreme points, and lay against them two straight-edges, so that their intersection shall coincide with the central point. Secure the straight-edges in this position with a cross-piece, as shown in fig. 15, and the curve

Fig. 15.

may then be drawn with a fine pointed pencil placed at the intersection of the rules, the trammel being pressed steadily against the pointers whilst the curve is drawn. Take off the superfluous wood with a plane, and the sweep is ready for use.

An instrument called a cyclograph constructed on this

principle is sometimes used for drawing arcs of circles, but it is expensive; and the use of sweeps is preferable, if the length of the curve is such that the work cannot be done without shifting the instrument, as it is very difficult to make a neat junction between the different portions of the curve.

A method of calculating the position of a number of points in a curve of which the radius is known, will be found in art. 84.

65. *Scales*.—Drawing scales are made of brass, ivory, box-wood, and cardboard. They are divided in a variety of ways, some being covered with divisions, whilst others are divided at the edge only. Those of the latter kind are called plotting scales, and are preferable to the former, as the dimensions can be pricked off at once on the paper along the edge of the scale, whilst the others require them to be transferred from the scale to the paper with compasses, an operation which tends to deface the scale, and introduces a chance of error, which it is well to avoid.

The engine-divided card-board scales, manufactured by Holtzapffel and Co., possess many advantages, of which the principal ones are, their extreme accuracy and their low price. They are sold at 9s. the dozen; and, although made of perishable material, will last many years. Box-wood plotting scales 12 in. long are usually sold at about 4s., and ivory scales of the same length at about 10s.

66. Before commencing a large drawing, it is advisable to cut a strip from the edge of the paper, and to make upon it a scale of the whole length of the intended drawing. The use of a scale of this kind saves much time that would otherwise be spent in setting off, and checking long dimensions by numerous applications of

a comparatively short scale; and, the scale being kept rolled up with the drawing, will generally contract and expand with it, and thus obviate the perplexing difficulties which arise from the expansion and contraction of the paper from atmospheric changes.

Independently, however, of the constant variation which is daily taking place with every change in the weather, all paper is subject, when worked upon, to a certain amount of permanent contraction, which must be allowed for in making the paper scale. The amount of this correction in the scale must depend upon the seasoning the paper has received, and the texture of the paper itself, so that no precise rule can be given for it. During many years' observation of parish and railway plans, we have found it vary from $\frac{1}{2000}$ to $\frac{1}{4000}$, and a mean between the two may be safely taken; that is, the length of each foot on the scale should be 1.003 ft. After a very few days' work the warmth of the hand will cause the paper to shrink to the correct length, or nearly so.

67. Both box-wood and ivory scales are subject to expansion and contraction, but the amount of this is too trifling to be taken into account.

68. *Standard Scale*.—In order to insure uniformity in the dimensions of a large building, every master mason should keep a standard metal scale very accurately divided, by which all the scales used in making the working drawings, and the rods employed in setting out the work, should be carefully tested. Unless this is done it is very difficult to keep the work exact, particularly in erecting bridges of large span.

69. *Centre Lines*.—On commencing a drawing two centre lines at right angles to each other should be drawn through the middle of the work, of the whole

length and breadth of the paper. Lines parallel to these should be drawn in pencil at regular distances, corresponding to some even division of the scale, dividing the paper into squares or rectangles of convenient size. The intersections of the lines should be punctured with a needle, and marked in faint colour thus +, after which the pencil lines may be rubbed out.

This precaution is of great use in keeping the work perfectly true and square, as the divisions are a complete check on the parallelism of the lines of the drawing, and afford a ready means of drawing lines in a given direction, on any part of the paper, without the necessity of reference to the principal centre lines.

They also are of great use in ascertaining the exact amount of contraction which the paper may undergo from time to time, and in checking the distances from the centre lines.

70. *Figuring*.—The manner in which working drawings are figured is of considerable importance. The horizontal dimensions should be referred to centre lines marked on the *whole* of the plans, and the positions of all the principal points should be obtained in the execution of the work by direct reference to the centre lines, and not by measurement from intermediate points. This precaution confines any trifling error to the spot where it occurs, instead of its being carried forward through the work, as would otherwise be the case. To enable this to be readily done, two sets of dimensions will be required: 1st, the dimensions from point to point; and 2nd, those from the principal points to the centre lines. If any clerical error be made in figuring any of the dimensions, it can by this means also be detected and corrected, as every leading dimension is given once in gross, and can be also obtained by addi-

tion in two other ways. In spite of the utmost care errors will creep into the working drawings, and those who have lost valuable time through some apparently trivial mistake in a figure, can appreciate the advantage of being able to *correct* mistakes as well as to detect them.

71. Elevations and sections should be figured on the same principle as the plans, vertical lines corresponding to the centre lines of the latter being marked upon them whenever practicable*.

The vertical heights should all be referred to a common datum line, which should coincide, if possible, with some leading line in the design. In the execution of the work, the height of the datum line should be permanently marked by a stout stake driven firmly into the ground at the proper level.

72. It generally happens in the execution of large works that their levels require to be determined with great precision. Before making the working drawings therefore, it is always advisable to put down a permanent mark at the intended site, and to ascertain its height with reference to the levels of the proposed works. In figuring the elevations and sections, the position of the datum line with reference to this mark must be accurately noted, and there will then be no difficulty when commencing operations in ascertaining the proper level at which to start the work.

73. *Copying Drawings.*—To make a correct duplicate of a large drawing is a work of some difficulty. The most correct method is to draw the whole afresh to scale, but this is very tedious. Two methods are in use for abridging the labour of the draughtsman. One is

* This is done on the assumption that the work is intersected by vertical planes passing through the centre lines of the plans.

to lay the drawing *over* the blank paper, and to prick through the leading points with a needle. The copy is then easily lined in between the points thus formed. The other method is to place a sheet of transparent paper over the drawing, and having secured the two together, so as to prevent all possibility of their shifting, the copy is drawn on the transparent paper.

Both these methods possess the common defect of producing a copy of the original, not of exactly the same size, but from the shrinking of the paper a little smaller, and in consequence the real scale will be less than the nominal one. And this is not the only evil, for in a large drawing the contraction of the paper is often so irregular, that the straight lines become twisted more or less; and these irregularities becoming still more distorted in the copy, the latter is of little value. There is also great difficulty in pricking off a large drawing with accuracy, as it is difficult to get the paper to lie sufficiently flat for that purpose.

74. The method the author would recommend is, first, to divide the blank paper into squares or rectangles similar to those of the original; next, to make a careful tracing of the latter, marking the divisions of the squares; and, lastly, to lay this tracing on the blank paper, and to prick it through, adjusting the work in each square to the new lines. By this means the errors of shrinkage and distortion will be corrected, and the copy when quite finished will be of exactly the same size as the original. The tracings being carefully laid aside will serve for any number of copies that may be required.

75. *Platform Work*.—The laying down of the work to its full size on a platform is done by methods precisely similar to those in use for making large drawings on paper, except that all the instruments are on a larger

scale, and that the bradawl and chalk line take the place of the needle and silk thread.

To ensure accuracy and uniformity in the work, the rods used for setting off the dimensions should all be divided from the standard scale referred to in a previous article.

Great care should be taken to render the platform perfectly level and quite firm, so that there shall be no chance of any of the lines shifting their position.

LINEAR DRAWING.

STRAIGHT LINES.

76. To draw a straight Line between two given Points.—Insert a needle at each of the given points; press the straight-edge gently, but firmly against them, and draw the line with the pen or the pencil held against the straight-edge, so as exactly to range with the centres of the needles.

If the line to be drawn be of considerable length, say 15 ft. or 20 ft., so that it cannot be drawn with the straight-edge at one operation, the silk thread must be used as follows:—

Insert the needles at the extremities as before, and strain the silk tightly between them; puncture the paper in the line of the thread at short intervals, and draw the line in between the points thus founded as before.

This method should be always resorted to where extreme accuracy is required. A common but vicious mode of drawing long lines is to produce them with the straight-edge until they are of the required length; but this method is not susceptible of minute accuracy.

77. To draw straight Lines parallel to a given straight

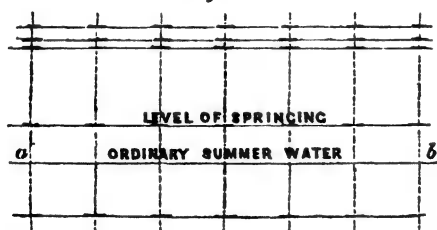
Line.—If the lines to be drawn do not exceed 2 ft. in length, they may be drawn by placing the working edge of a large set-square to coincide with the given line, and fixing a straight-edge against the bottom of it, keeping it steady with two needles and a weight or two if necessary. All the lines drawn with the set-square will of course be parallel to each other. If the lines to be drawn are parallel to either of the centre lines, nothing more will be required than to set the straight-edge to the nearest divisions of the paper.

If the lines are very short, a small set-square and straight-edge may be used, the latter being steadied with the left hand, whilst the set-square is moved, and the lines drawn with the right hand.

For short lines also, the parallel ruler is much used by professional draughtsmen, but it requires a practised hand to ensure perfect accuracy in its use, and we have not, therefore, mentioned it previously.

Long lines must be drawn with the straight-edge through points previously marked off. Let it be required, for instance, in making an elevation of a bridge to draw a series of lines parallel to the line *a b*, fig. 16, which we will suppose to be 20 ft. long.

Fig. 16.

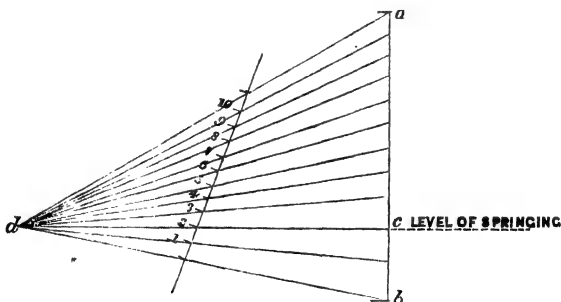


Erect perpendiculars to *a b* at such distances apart that the straight-edge will extend over three divisions or more, and on these perpendiculars set off by scale the exact distances from *a b* at which the parallel lines are to be drawn. This is best done by setting off the distances on a strip of paper, and pricking them off on each per-

pendicular. The lines can then be drawn through the points thus found with great accuracy, as the slightest error in any part of a line is at once detected by reference to the more distant points.

78. *To divide a straight Line into a given Number of unequal Parts, which shall diminish in regular Progression, and so that a given Division shall pass through a given Point.*—Let $a b$, fig. 17, be the height of the pier of a bridge, which it is proposed to divide

Fig. 17.



into eleven quoin, the top of the second quoin being required to coincide with c , the level of the springing of the arch. Assume any convenient point d , and join $a d$, $c d$, $b d$; take a slip of paper, divide its edge into eleven equal parts of convenient size, and slide it over the triangle until the zero, and the 2nd division, are respectively on the lines $b d$, $c d$, whilst the last division is on the line $a d$. Prick off the points 1, 3, 4, 5, 6, 7, 8, 9, 10, and draw lines through them, intersecting the line $a b$, which will then be divided as required.

The method of arranging the sizes of the courses of a building, so that the first and last shall be of given heights, is precisely similar.

The above is a very convenient practical rule, but can only be applied within certain limits.

ANGLES.

79. *To set off a right Angle.*—There are three ways of doing this in common use. It may be done on a small scale mechanically with a straight-edge and set-square. On a large scale it may be performed by describing with beam-compasses a triangle, of which the sides are respectively as 3, 4, and 5; or by describing two isosceles triangles on a common base, of which the centre is the point through which the perpendicular is to pass, see fig. 18, plate 1. This last method is the most perfect of the three, as the accuracy of the work is at once checked by trying with a silk thread whether the vertices of the triangles range with the centre of their common base.

80. *To set off an acute Angle.*—This may be done on a small scale by pricking off the angle from the edge of a protractor, but this method is inapplicable to large drawings, as the sides of the angle would have to be *produced* from a line probably not exceeding a few inches in length.

The best method of setting off an angle, of which the sides are of considerable length, is to describe with beam compasses an isosceles triangle of which the base and sides are respectively as the chord and radius of the angle. The length of the chord is obtained as follows: since the chord of any arc is double the sine of half the angle subtended by that arc, we can find the chord for any angle, by taking from a table of natural sines the sine of half that angle and doubling it*. Tables of natural sines are calculated for radius = 1, the lengths of the sines being given in decimals; in plotting an angle by this means it

* Instead of *doubling* the sine, we may use *half* the radius, which is a much simpler plan, although the principle is not so immediately apparent.

is, therefore, necessary that the scale should be divided decimally, and that the radius chosen should be ten, or some multiple of that number.

Example.—To set off an angle of 70° , the sides to be not less than 8 ft. long. Look in the table for the natural sine of 35° , which is .5735764. The length of the chord will be twice this, or 1.1471528. Taking the radius in inches, the nearest convenient number will be 100, and accordingly the decimal point must be shifted two places, making the length of the chord 114.71528 inches.

It is always desirable in plotting angles, that the points found should be *beyond* the work, and not within it, so that there may be no necessity for *producing* their sides.

81. *An Obtuse Angle* is plotted by producing one of the sides and setting off the supplement of the required angle.

82. *Measurement of right-angled Triangles.*—In any right-angled triangle, if one side, and one of the acute angles be given, the remaining sides can be readily found by calculation, with the help of a table of sines, cosines, secants, and tangents. We presume the reader to be familiar with the method of doing this, but it may be useful here to insert the formulæ.

In the right-angled triangle abc , fig. 19, plate 1. Let $\angle abc$ be the given angle—the $\angle bac$ will of course be its complement.

1st, Let the hypotenuse ab be the given side.

$$\text{then side } ac = ab \times \sin \angle abc$$

$$\text{and side } bc = ab \times \cos \angle abc.$$

2nd, Let the given side be one of those containing the right angle, as bc .

$$\text{then side } ab = bc \times \sec \angle abc$$

$$\text{and side } ac = bc \times \tan \angle abc.$$

Deducting this last quantity from 164 ft., we find the remainder $be = .291$ ft.

To set off the required distance on the curve, set off the chord $ab = 153.718$ ft., with beam-compasses, and from b set off $be = .291$ ft.; the length of the arc abe will be 164 ft., as required.

90. It is often necessary to transfer the divisions of the arch-stones from the development to the elevation of an arch. The best way of doing this is to set them off from the development on a long lath, and to bend the latter round the curve in the elevation, to which the divisions can then be readily transferred. This method confines any little inaccuracy to the joint where it occurs; but if it be attempted to set off the joints stone by stone with compasses, great difficulty will be experienced in making the minute allowance which is necessary for the difference between the length of the curve included between two joints and the corresponding chord, which is the distance to which the compasses must be set.

91. *Method of describing an Ellipse.*—On a small scale, and where it is desirable to avoid defacing the paper with the points of the compasses; as, for example, in drawing the coping of a curved wing-wall, the simplest mode of proceeding is to find a number of points in the curve, and to connect them by means of a curved ruler, the edges of which are cut into a continuous series of curves of different radii.

* Any number of points in an ellipse may be found as follows:—Let af , ef , fig. 26, plate 1, be the respective semi-diameters of the ellipse. With f as a centre and af and ef as radii, describe two quadrants. Divide the larger quadrant into any convenient number of divisions, as 1, 2, 3, and draw the lines $1\ 1_1f$, $2\ 2_1f$, $3\ 3_1f$, cutting the lesser quadrant at 1_1 , 2_1 , 3_1 . From the points 1 and 1_1 , draw lines parallel to the diameters cutting each

other at b , then b will be a point in the ellipse. In a similar manner will be found the points c and d .

92. When an ellipse has to be drawn on a large scale, the best way is to strike it from centres; and, although this is only an approximation, no portion of an ellipse being a circular curve, no appreciable error will result if a sufficient number of centres be taken.

The following method is very simple. Having found a number of points in the curve as b, c, d , draw the chords ab, bc, cd, de . Bisect de with a perpendicular cutting fe produced in g ; then g will be the centre for the portion of the curve between e and d . Join dg and bisect cd with a perpendicular cutting dg in h ; then h will be the centre, for the portion of the curve between c and d . The centres i and k are found in a similar manner.

93. To set out an ellipse on a platform; when the scale is such that the operation must be performed without making use of centres, we must proceed rather differently.

Divide the right angle contained between the two semi-diameters into any convenient number of angles, as $af1, af2, af3$, fig. 27, plate 1, and multiply their respective sines and cosines, the former by radius ef , and the latter by radius af . This will enable us to lay down the points b, c, d , by means of offsets from the diameters, as shown in the figure.

The curves ed, dc , &c., must be drawn in with curved rules, made as directed in article 64.

To find the radii, draw the chords ed, dc , &c.; bisect the angles formed by their intersections with short lines as shown in the figure. On these bisection lines, let fall perpendiculars, as dd_1, cc_1 , &c., and the several radii can then be calculated as in article 83.

94. An ellipse of moderate size can also be struck on a platform, from the foci, as follows:—

From e as a centre, fig. 28, plate 1, with radius af , describe arcs cutting aa in m, l , which will be the foci of the ellipse. Put in a brad-awl at each of the foci, and round them pass an endless cord of such length that, when strained tight, it will just reach the point e . The curve may then be drawn in with a brad-awl or a drawing knife pressed firmly against the cord.

This is a very expeditious method; but it requires considerable management to produce an even line, and is not susceptible of minute accuracy. The practical difficulty arises from the elasticity of the cord.

95. *To draw a Line perpendicular to the Circumference of an Ellipse at any Point*, as n , fig. 28, plate 1.—Join mn , nl : a line bisecting the angle $mn l$ will be perpendicular to the curve at n .

This problem is required in drawing the joints in the elevation of an elliptical arch.

96. *Spiral Curves*.—In making drawings of oblique bridges, numerous projections of spiral lines have to be drawn; and it is of importance that this should be done with great exactness. The best method of accomplishing this, is to make a very accurate template for each set of curves in card-board or veneer, which will ensure perfect uniformity in the work, and also save much of the draughtsman's time.

97. *Principles of Projection*.—The working drawings of the mason may be classed under two heads:—First, geometrical projections; and, secondly, developments of surfaces. The geometrical projections are always made on either horizontal or vertical planes; the drawing being called in the first case a *plan*, and in the second case an *elevation*. When the plane of projection cuts the object represented in a vertical direction, the drawing is called a sectional elevation, or, in brief, a *section*. It

will be observed that most plans of buildings are, in fact, horizontal sections, but the term is technically applied to vertical projections only. Developments are representations of the surfaces of solids, as they would appear if unwrapped and laid flat, and are made use of to obtain the dimensions of surfaces which, from their inclined position, become foreshortened both in plan and section; and for the delineation of curved surfaces, which cannot be accurately represented in any other manner.

The nature of plans and elevations may be clearly understood by considering them as perspective projections on a sphere of infinite radius of which the centre is the point of sight.

98. The following properties of geometrical projections should be kept in mind.

Lines.—All horizontal lines will be represented of their true length and curvature on plan.

All vertical straight lines will be represented of their true length in elevation.

All lines inclined to the horizon will be more or less foreshortened in plan.

99. The length of any inclined straight line may be obtained from the plan and elevation by a simple construction. Thus to find the lengths of the arrises of a square pyramid: let ac , fig. 29, plate 1, be the vertical height of the pyramid, and cb the half diagonal of the base; then the required length ab is the hypotenuse of the right-angled triangle acb , and can be formed by constructing the triangle and measuring the hypotenuse, or by calculation, since $ab = \sqrt{ac^2 + bc^2}$.

100. *Surfaces.*—Horizontal planes will be represented by identical figures on plan, and by straight lines in elevation. Thus the plan of a circle parallel to the horizon will be a circle, and its elevation will be a straight line;

if inclined to the horizon, its plan will be an ellipse in all positions except the vertical, when its plan will be a straight line, and its elevation a circle, a straight line or an ellipse, according to the position of the plane of elevation.

101. *Solids*.—The plan of a right cone standing on its base will be a circle, and its elevation a triangle.

The plan of a right cylinder, similarly placed, will be a circle, and its elevation a rectangle.

The plan and elevation of a sphere will always be circles.

Figs. 30, 31, 32, 33, and 34, explain the manner of projecting the plan and elevation of the prism, pyramid, cone, cylinder, and sphere.

Fig. 30.

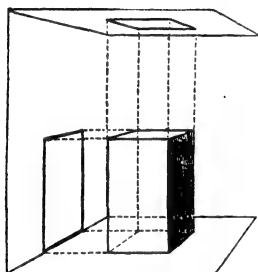


Fig. 31.

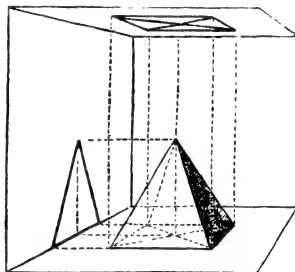


Fig. 32.

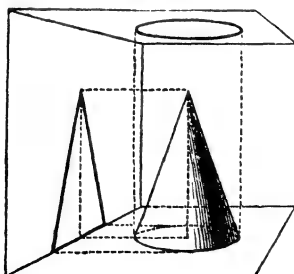


Fig. 33.

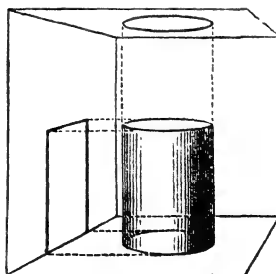
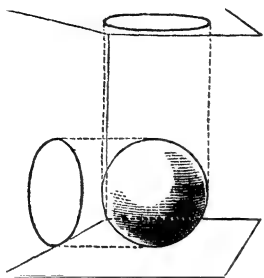


Fig. 34.



102. *Sections.*—The following properties of the cone, cylinder, and sphere, should be borne in mind :—

Every plane section of a cone perpendicular to its axis will be a circle.

Every plane section of a cone passing through the vertex and the base will be an isosceles triangle.

Every plane section of a cone cutting its axis at an acute angle, greater than that made by the slant side, will be an ellipse, or a segment of one.

Every plane section of a cylinder parallel to its axis, will be a rectangle.

Every plane section of a cylinder perpendicular to its axis will be a circle.

Every plane section of a cylinder cutting the axis obliquely will be an ellipse, or a segment of one.

Every plane section of a sphere will be a circle.

103. The sections above enumerated can be projected in any position with very few lines ; the projection of an ellipse being always either a straight line, a circle, or an ellipse, and the only data required for drawing the latter figure are the lengths of the major and minor axes. There are, however, many other curves, such as those formed by the intersection of two curved surfaces, which are not so easily described, and which require a considerable amount of projection, and transference of lines, in order to represent them accurately.

104. *Developments.*—The curved surfaces of solids may be classed under two heads ; 1st, those with which

a straight-edge will coincide in one direction, as the surfaces of the cone and cylinder; and 2nd, those with which a straight-edge will not coincide in any direction, as the surface of a sphere. The former are sometimes called curved planes, and their development, in the case of the cone and the cylinder*, is very simple. The latter can only be developed approximately, because it is impossible to bend a plane, so as to coincide with a spherical, surface.

105. The development of the curved surface of a right cone will be a sector of a circle, whose radius is the slant height of the cone; the length of the arc being equal to the circumference of the base of the cone; see fig. 35.

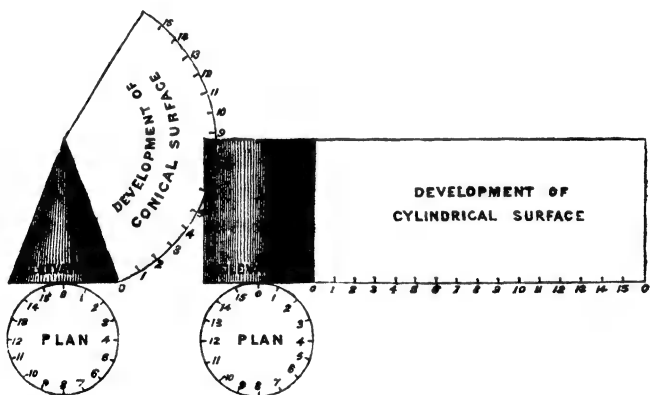


Fig. 35.

Fig. 36.

106. The development of the curved surface of a right cylinder will be a rectangle, whose length is the axial length of the cylinder, and whose width equals the circumference of its base; see fig. 36.

* *Winding* surfaces cannot be developed even approximately, being convex in one direction, and concave in the opposite.

107. The surface of a sphere may be developed approximately in three different ways; 1st, it may be considered as a polyhedron, of which each side will be a plane surface; 2nd, it may be divided into gores like the gores of a balloon, in which case each gore will be a portion of a cylindrical surface; lastly, it may be divided into zones, each of which may be treated as a portion of a conical surface. This last method is the one most practically useful, and will be understood by inspection of fig. 37, plate 4.

PROJECTIONS OF THE CONE.

108. The several projections of the cone which we are about to describe are principally required by the mason in the execution of battering walls, on a curved plan, which form portions of hollow cones. The projections and development of a right cone have been explained above, in arts. 101 and 105.

109. *To draw the Projections of an inverted Cone from which an oblique Frustrum has been removed.*—In fig. 38, plate 4, side elevation, let bde be the inverted cone, and bdm the frustrum removed. Bisect bm in n , and through the point n draw $e_1no b_1$ parallel to the base, and cutting the axis of the cone at e_1 . Draw bb_1 parallel to the axis of the cone, cutting $e_1no b_1$ in b_1 , and making e_1b_1 equal to cb , the radius of the base. It may be easily shown that $e_1n = ob_1$. In the *plan* draw the diameters bed and aec perpendicular to each other, so that all straight lines drawn on the plane of intersection, parallel to aec , shall be horizontal. Set off en , bo respectively equal to e_1n , ob_1 in side elevation. With radius eo , and centre e , describe the quadrant oqr , and through n draw pnq parallel to ac , cutting the arc oqr in q , and make $np = qn$.

Set off $nm = nb$; then bm and qp will be respectively the major and minor axes of the ellipse, which is the horizontal projection of the oblique section of the cone.

Since by construction $nb = eq$, the length of the semi-axis minor $qn = \sqrt{nb^2 - en^2}$. In ordinary cases, the difference of the lengths of the major and minor axes is so small, that the quarter ellipse may be drawn without sensible error, as a circular curve, with radius nb and centre on qp , removed from n by the difference between the semi-axes.

Fig. 39.

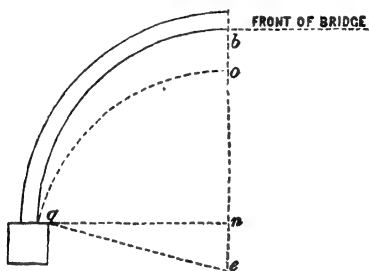
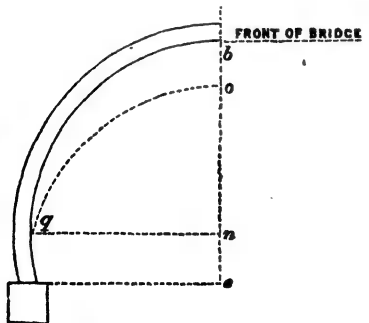


Fig. 40.



110. It will be seen by inspection of fig. 39 that in building a curved wing-wall, terminating in a pier as there shown, the horizontal distance bn (fig. 39) should not exceed bn in fig. 38, plate 4: or the coping would have a very unpleasant appearance, as shown in fig. 40.

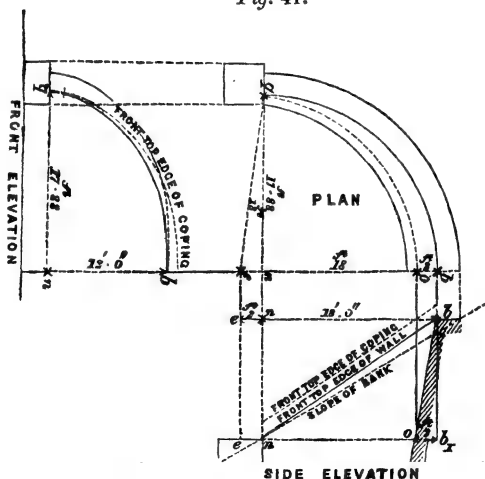
111. When the plan of the coping is less than a quarter ellipse, the sides of the pier must be made square with a tangent to the ellipse at the point of intersection with the wing-wall.

112. *To draw the front Elevation.*—Set off $bn = b$

in side elevation, and $nm = bn$. Through n draw qnp parallel to ac , and set off $nq = nq$ in plan; and $np = nq$. Then qp , bm are respectively the major and minor axes of the ellipse $bqmp$, which is the vertical projection of the oblique section of the cone.

113. In drawing curved wing-walls to support an embanked approach to a bridge, the data given or assumed are the height $b b_1$; the inclination of the slope of the bank which should coincide with that of the top of the wall*; and the batter or slope of the face of the wall. As we have often found beginners to be very much at a loss how to draw the plan and elevation of such a wall without covering the paper with unnecessary lines, we subjoin an example.

Fig. 41.



* The coping of a wing-wall is sometimes made to stand up above the slope of the bank, but this has an awkward appearance. To make the top of the wall form a spiral plane, as recommended in "Nicholson's Railway Masonry," is, perhaps, the worst plan that can be adopted, as the coping is not parallel to the slope of the bank.

To avoid confusing the diagram the coping of the wall is omitted.

Let $b b_1 = 12$ ft.

Let the slope of the top of the wall be $1\frac{1}{2}$ horizontal to 1 vertical.

Let the batter of the face of the wall be 1 horizontal to 6 vertical.

Then $n o b_1 = 12$ ft. $\times 1\frac{1}{2} = 18$ ft.

and $e n = o b_1 = \frac{12}{6}$ ft. = 2 ft.

Transfer these dimensions to the plan.

$n b = e o = e q = 18$ ft.

$q n = \sqrt{n b^2 - e n^2} = \sqrt{324 - 4} = 17.88$ ft.

The *plan* of the front line of the top of the wall will therefore be a quarter ellipse, whose semi-diameters are respectively 18 ft. and 17.88 ft.

The *Front Elevation* of the front line of the top of the wall will be a quarter ellipse, of which the semi-diameters are respectively 12 ft. and 17.88 ft.

114. *Development of the Cone*.—Divide the circumference of the base into any convenient number of equal parts as shown in the plan, fig. 38, plate 4, by the points f, g, h , &c., and transfer these divisions to the side elevation. From the new points thus found draw lines radiating to the apex of the cone, cutting $b m$ in f_1, g_1, h_1 , &c. Through f_1, g_1, h_1 , &c., draw lines parallel to the base, and cutting the sides of the cone. Having drawn the development of the slant surface of the cone, divide the arc $b d b$ to correspond with the divisions of the base in plan, and draw the radiating lines $f e, g e, h e$, &c., corresponding to the radiating lines in the side elevation.

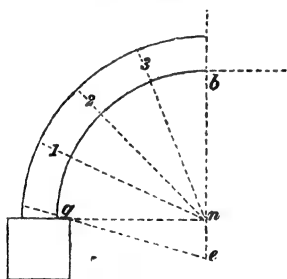
From the points found on the slant side of the cone, with e as a centre, draw circular arcs cutting the radiating lines $f e, g e, h e$, &c., in f_1, g_1, h_1 , &c. A curve drawn through these last points will be the develop-

ment of the line bounding the oblique section of the cone.

The oblique section of the cone will form an ellipse, whose major axis $= b m$ in the side elevation, and whose minor axis $= q p$ in the plan.

115. *To project the Lines of the Coping of a curved Wing-Wall.*—The coping of a curved wing-wall is worked in such a way that its bed shall be every-

Fig. 42.



where level in a direction perpendicular to the curve of the wall. Thus, fig. 42, any number of lines as 1, 2, 3, drawn perpendicular to the curve of the wall, will be horizontal lines. If the coping bed were made level in the direction of the centre of the cone, as shown by the line $e q$, it is evident that the intersection of the

coping with the pier will not be a level line, the front of the coping being higher than the back, which would have a most unsightly appearance.

116. If the top of the wall be worked as above described, the front and back edges will lie in *planes** of different inclinations, intersecting each other on the line $q n$. It is usual to make the *back* of the wall coincide with the slope of the bank. The front line will therefore be found in side elevation, without any transference of lines, by setting off the width of the top of the wall

* If the front and back edges of the top of the wall are made to lie in *planes*, so as to be represented in side elevations by straight lines, all level lines in the coping bed will be curved, and not straight; but the curvature is too small to be measured in so short a distance, and cannot be distinguished from a straight line.

from the top of the slope as shown in fig. 41, and drawing a straight line from the point thus found to n . In front elevation all the lines of the coping will be elliptical curves.

117. It may be necessary to remark that the top and bottom lines of the coping in side elevation will not be parallel to each other. This arises from the thickness being set off at the top in a vertical, and at the bottom in an inclined direction, so that the lines will diverge from the top downwards. (See fig. 41.)

118. *Intersection of a Cone and a Cylinder*, fig. 38, plate 4.—This is a problem of not unfrequent occurrence, as in the case of a cylindrical culvert passing through the curved wing-wall of a bridge. In the diagram here given the axes of the cone and cylinder are made to intersect each other at right angles.

In front elevation the projection of the intersection of the cylinder with the slant surface of the cone, will be a circle. Draw two diameters, the one parallel to the base, the other to the axis of the cone. Divide the circumference of the circle into any convenient number of equal parts, as 16. Divide one of the diameters into 8 parts by perpendiculars drawn through the divisions on the circumference, and transfer these divisions to the axis of the cone in side elevation, as shown at a_2, b_2, c_2 , &c. Through these points draw lines parallel to the base, and cutting the slant side of the cone in 1, 2, 3, &c. Transfer the divisions on the diameter of the cylinder to the diameter ac in plan, and through the points thus found draw the perpendiculars aa_2, bb_2 , &c. Draw the diameter bed at right angles to aec , and set off on ed the divisions $e1, e2$, &c, respectively equal to the corresponding lengths $1a_2, 2b_2$, &c., in the side elevation. Through the points thus found, with centre e

draw circular arcs cutting the perpendiculars just drawn in a, b, c, d , &c. A curve traced through these points will be the *plan* of the curve formed by the intersection of the cone and cylinder.

In side elevation make $a a_2, b b_2$, &c., respectively equal to $a a_2, b b_2 c c_2$ &c., in plan; and a line drawn through the points a, b, c, d , &c., will be the side elevation of the curve of intersection.

The development of the curve of intersection on the surface of the cone is found by transferring the distances $e m, e 1, e 2$, &c., on the slant side of the cone, in the side elevation, to the corresponding line in the development, and through the points thus formed drawing with the centre e , circular arcs, $a q, b p, c o$, &c., respectively equal to the corresponding arcs, $a q, b p$, &c., in plan.

To find the development of the surface of the cylinder draw the straight line $m_1 m_1$ equal to the circumference of the right section of the cylinder, and divide it into the same number of equal divisions. At the points, a_1, b_1, c_1 , &c., thus found, erect perpendiculars, $a_1 a, b_1 b, c_1 c$, &c., respectively equal to the corresponding lines, $a_1 a, b_1 b$, &c., in the side elevation. The curved line, $m a b c$, &c., drawn through the points thus found, will be the development of the curve of intersection in the surface of the cylinder.

PROJECTIONS OF THE CYLINDER.

119. The projections and development of the cylinder have been already described in arts. 101 and 106; but, as it is of great importance that the subject should be thoroughly understood, we return to it again for the purpose of explaining the nature of spiral lines, and the manner of projecting them.

120. In the diagram, fig. 43, plate 5, the right cylinder is supposed to be in a horizontal position, in order that the application of the projections here described to the construction of vaults and arches may be more clearly understood.

The elevations of the ends of the right cylinder, $A B C D$, fig. 43, plate 5, will be circles exactly coinciding with the square sections. The plan will be a rectangle, and the development, $B A B C D C$, will also be a rectangle, whose width, $B A B$, = circumference of the circle formed by the square section.

121. If the cylinder be cut obliquely by a plane surface, as shown by the line $E B$ on plan, the resulting section will be an ellipse, whose major axis = $E B$, and whose minor axis = diameter of the cylinder.

The development of the curve of the oblique section is found as follows: divide the circumference of the square section into any convenient number of parts, as sixteen. Divide the width of the development in the same manner as shown at 1, 2, 3, &c. Transfer the divisions on one-half of the square section to the plan as shown at $c\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ d$. Through the points thus found, draw lines parallel to the axis of the cylinder cutting the line $B E$ at $a\ b\ c\ d\ e\ f\ g$. Through the points 1 2 3 4 5, &c., in the development, draw lines 1 a , 2 b , 3 c , &c., parallel to the side of the cylinder, and respectively equal to the lines 1 a , 2 b , 3 c , &c., in plan. A curve drawn through the points $a\ b\ c\ d$, &c., will be the development of the curve of the oblique section.

122. If we draw on the development any straight line in an oblique direction, as $C E B$, this line, when wrapped round the surface of the cylinder, will form a spiral line whose inclination to the base of the latter will be uniform throughout its whole extent.

123. In building cylindrical arches on an oblique plan in spiral courses, the lines of the coursing joints are called *coursing spirals*; and those drawn perpendicular to them, for the purpose of determining the position of the heading joints, are called *heading spirals*.

124. Let it be required to project a spiral, as $C E B$, which makes one revolution in the length $C B$. Having divided the plan and development, to correspond with the divisions on the circumference of the square section as before described, join $C B$, and this line will be the development of the spiral $C E B$.

Make the lengths $1 a_2, 2 b_2, 3 c_2$, &c. on plan respectively equal to the lengths $1 a_2, 2 b_2, 3 c_2$, &c. on the development. A curved line drawn through the points a_2, b_2 , &c., will be the horizontal projection of the spiral $C E B$.

125. *In the Elevation of the Face of an oblique cylindrical Arch, to draw the spiral Lines in the Soffit, as, for example, the heading Spiral $B a_1 b_1 c_1 d e_1 f_1 g_1 E$ in the Plan.*—The plan and development of the spiral are found as above described. Draw $E B = E B$ in plan. Bisect it in d_1 , and on $E d_1 B$, with d_1 as a centre, draw the square section of the arch, divide it into eight equal parts, as before done to obtain the development of the cylinder, and through the opposite divisions, 1 7, 2 6, 3 5, draw lines parallel to $E B$. From the points a_1, b_1, c_1 , &c., in plan, let fall perpendiculars on $E B$, and transfer the points thus formed to $E B$ in elevation. Erect perpendiculars at these points, cutting the lines 1 7, 2 6, 3 5, in a_1, b_1, c_1 , &c., and a line drawn through these points will be the elevation of the spiral projected on a plane parallel to that of the face of the arch. The elevation of a coursing spiral is obtained in the same way.

126. *To draw an oblique semi-cylindrical Arch with*

If any two sides are given, the third side may be found arithmetically in the absence of a table of sines.

If the hypotenuse be one of the given sides, then side $a c = \sqrt{a b^2 - b c^2}$, and side $b c = \sqrt{a b^2 - a c^2}$.

If the two sides containing the right angle be given, then side $a b = \sqrt{a c^2 + b c^2}$.

The solution of right-angled triangles is very fully explained in Mr. Heather's "Treatise on Mathematical Instruments,"* to which we would refer the reader who is not familiar with the subject: the foregoing cases are merely inserted here to assist the memory.

CURVED LINES.

83. *Circular Curves.*—The following problems will be found useful.

Given the Span and Rise of a circular Arc to find the Radius.

Let r = radius.

s = half-span.

v = rise, or versed sine.

$$\text{Then } r = \frac{s^2 + v^2}{2v}.$$

Demonstration (fig. 20, plate 1).—Let $a d e$ be the arc of which the radius is required; $a b$ the half span, and $b d$ the rise, and let c be the centre of the circle.

Join $a c$, $d c$ and $a d$; bisect $a d$ in f and join $f c$. The right-angled triangles, $b a d$, $f c d$, are similar, having the common angle $f d c$;

$$\text{therefore, } b d : d a :: f d : d c = \frac{f d \times d a}{b d}.$$

* "A Treatise on Mathematical Instruments." Rudimentary Treatises, 2nd Series. Weale. Lond. 1849.

$$\text{But, } fd = \frac{da}{2}.$$

$$\therefore dc = \frac{\frac{da}{2} \times da}{bd} = \frac{da^2}{2bd} = \frac{ab^2 + bd^2}{2bd} = \frac{s^2 + v^2}{2v}.$$

Q.E.D.

84. *The Radius being given, to find the Length of an Offset at any given Point on a tangent Line.*

Let r = radius.

t = distance on tangent line from the point of contact.

o = offset.

$$\text{Then } o = r - \sqrt{r^2 - t^2}.$$

Demonstration (fig. 21, plate 1).—Let ae be the given tangent, c the centre of the circle, and eb the offset, of which the length is required. Join ac , bc , and draw bd parallel, and by construction, equal to ae . Then $eb = ad = ac - dc$.

$$\text{Now, } dc = \sqrt{cb^2 - db^2}$$

$$\therefore eb = ac - \sqrt{cb^2 - db^2} = r - \sqrt{r^2 - t^2}. \quad \text{Q.E.D.}$$

85. In designing large works it is often requisite to connect two straight lines by a circular curve. Before the offsets can be calculated for this purpose the following data must be known, viz., the angle formed by the lines to be connected, the radius of the curve, and the distance from the point of intersection to the points of contact. The first of these conditions is generally determined by the circumstances of the case; with regard to the second and third conditions, one of the two must be assumed and the other calculated from it.

86. First Case.—*The Distance of the Points of Contact from the Point of Intersection being given, to find the Radius.*

In the lines to be connected, let b and d (fig. 22, plate 1) be the points of contact, which will necessarily be equidistant from the point of intersection.

Join bd ; bisect it at e , and join ae ; then

$$\text{radius} = \frac{be \times ab}{ae}$$

Demonstration.—Let c be the centre of the circle; join bc and ec . The right-angled triangles abe and acb are similar, having the angle bac common to both;

$$\therefore ae : be :: ab : bc = \frac{be \times ab}{ae}. \quad \text{Q.E.D.}$$

The construction made use of in the above problem is useful for determining the radius of curvature of a wing-wall of a bridge.

Thus (fig. 23, plate 1), let df be the front of the bridge, d the point at which the curve is to commence, and b the point at which the wing-wall is to end. Join bd ; bisect it at e , and erect the perpendicular ea cutting df produced in a ; join ab , and calculate the radius as above.

87. Second Case.—*The Radius being given, to find the Distance of the Points of Contact from the Point of Intersection.*

To do this, assume any approximate points, as $b_1 d_1$ (fig. 24, plate 1), and find the corresponding radius $b_1 c_1$.

Let r = given radius = bc ,

r_1 = assumed radius = $b_1 c_1$,

t = required length on tangent line = ab ,

t_1 = assumed length on tangent line = $a b_1$,

then, $r_1 : t_1 :: r : t = \frac{t_1 r}{r_1}$.

88. *To find the Length of a circular Arc.*—If the radius

is not known, it may be found as described in art. 83. Let abe , fig. 25, plate 1, be a portion of the circumference of a circle, of which the radius $= r$. Assume any convenient angle, as acb , and calculate its chord as in art. 80. Set it off on the curve with beam-compasses, and measure the remainder, be , as a straight line, which may be done without sensible error, by assuming such an angle as will leave a very small remainder.

The semi-circumference of a circle is equal to radius $\times 3.1416$; the $\frac{1}{180}$ th part, or that corresponding to a single degree, is therefore equal to radius $\times .017453$. If we call n the number of degrees in any angle, acb , we have for finding the length of any arc, ab , the simple formula. Length of arc $= nr \times .017453$.

Example.—Let $r = 134$ ft. On examination let it be found, that the number of degrees which will give the smallest remainder is 70. The length of the arc, ab , will therefore be $70 \times 134 \text{ ft.} \times .017453 = 163.70914 \text{ ft.}$; to which must be added the remainder be , the sum of the two making up the whole length of the arc abe .

89. This problem is of great service in ascertaining the length on soffit of an arch of known span and rise, either for the purpose of dividing the arch-stones, or for laying down a development of the soffit.

Its converse is equally useful in setting off on a circular arc, a distance equal to a given straight line. Let it be required on the curve abe , fig. 25, plate 1, to set off a portion, ae , that shall be equal to a given straight line, say 164 ft. long.

Let the radius of the curve be 134 ft. as before, then

$$\frac{164}{134 \times .017453} = 70.14. \quad \text{Rejecting the decimals,}$$

find the chord of 70° , which for radius $= 134$ ft. is 153.718 ft., and the length of the arc $ab = 163.709$ ft.

a curved Face, fig. 44, plate 6.—Draw the square section and divide the soffit into any convenient number of equal parts, as eight. Transfer these divisions to the plan as shown in the diagram; and through the points 1, 2, 3, 4, 5, 6, 7, draw lines parallel to the springing lines of the arch $a a_1, i i_1$, cutting the face of the arch at $b c d e f g h$.

To develop the soffit, draw $i_1 a_1$ = the length of the soffit on the square section; and, having divided it into the same number of equal parts, set up the perpendiculars, $i_1 i, 7 h, 6 g, 5 f$, &c., respectively equal to $i_1 i, 7 h, 6 g$, &c., on plan. A curve drawn through $i h g f e$, &c., will be the development of the front line of the soffit.

To develop the face, draw $a i = a i$ in plan, and set off $a 1, 1 2, 2 3$, &c., respectively equal to $a b, b c, c d$, &c. Erect the perpendiculars $1 b, 2 c, 3 d$, &c., respectively equal to the heights $1 b_1, 2 c_1, 3 d_1$ in the square section, and a curve drawn through $a b c d e$, &c., will be the development of the face of the arch.

Cases similar to that here given are not of frequent occurrence, but they are sometimes unavoidable, as in building a skew culvert in the face of a curved wing-wall.

127. *Intersections of cylindrical Surfaces*.—The reader who has carefully studied the preceding pages will find little difficulty in applying the principles of projection to the delineation of the intersections of cylindrical surfaces. We shall, therefore, in the following examples of the intersections of vaulting surfaces, omit the detailed description of the manner of constructing the several projections and developments, trusting that the diagrams themselves will be found sufficiently explanatory.

128. Fig. 45, plate 6, represents the intersection of

two semi-cylindrical vaults of equal span. Each groin will form a straight line on plan, and its profile will be a semi-ellipse, whose semi-axis major $= a E$, and whose semi-axis minor $= D B$.

129. Fig. 2, plate 1, represents the intersection of a semi-cylindrical vault, $A B C$, with a cross vault, $A_1 B_1 C$ of smaller span, but of the same height, the groins being in vertical planes, and forming straight lines in the plan. In this case the square section of the smaller vault will be a semi-ellipse whose minor axis $= A_1 C$, and whose semi-axis major $= D B$. The profile of the groins will be elliptical, as in the last instance.

130. Fig. 3, plate 1, shows a method commonly adopted in the infancy of vaulting for constructing intersecting vaults of the same height, but of different spans. The smaller vault, as well as the larger one, was usually a semi-cylinder, and its springing was raised above that of the larger vault just so much as was required to make the crowns of the two vaults coincide.

By this awkward expedient, the necessity for which appears to have arisen from the builder's ignorance of the principles of projection, the groins are made to lie in twisted planes, and form waving lines on the plan. The groins themselves, when viewed from below, appear crippled, and have an unsightly appearance.

131. In fig. 46, plate 6, is shown the intersection of two vaults of different spans, springing from the same level. The groin thus produced is called a *Welsh groin*,

PROJECTIONS OF THE SPHERE.

132. We have already stated that the plan and elevation of a sphere will always be a circle; and that every plane section of a sphere will be a circle, the

projection of which will be a circle, an ellipse, or a straight line, according to its position. It is, therefore, unnecessary to say anything further here, either as to the projections or development of the sphere, beyond referring the reader to articles 101, 102, and 107, and to figures 34 and 37, plate 4, the latter of which illustrates the approximate development of a sphere, by considering it as a series of conical zones.

133. Fig. 47, plate 6, represents the intersection of a hemispherical dome, with four semi-cylindrical vaults, and will be understood without any verbal description.

134. If the reader has made himself master of the problems given in this section, he will have no difficulty in projecting the intersections of any curved surfaces whatever, of which the profiles and directions are given. We think it therefore unnecessary to swell the bulk of this little volume by any further examples, and proceed at once to the subject of the third section, namely, the application of masonic projection to the scientific operations of stone cutting.

SECTION III.

PRACTICAL STONECUTTING.

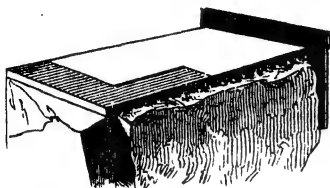
PART I.—GENERAL PRINCIPLES OF STONECUTTING.

FORMATION OF SURFACES.

135. In working a block of stone, the workman begins by bringing to a plane surface one of its largest sides, which will generally form one of the beds. Its required shape having been marked on the surface thus formed, either with the square or with a templet, chisel-

drafts are sunk across the ends of one of the adjacent

Fig. 48.

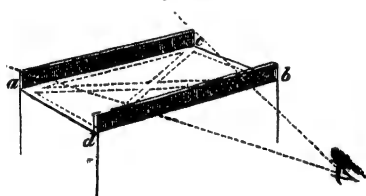


faces, by means of a square or a bevel, as shown in fig. 48, and this second face is worked between these drafts. The position of a third side is then determined, and its face worked in

the same manner, and this process is repeated until the block is brought to its required shape.

136. *To form a plane Surface.*—1st, when the surface is of considerable size.

Fig. 49.



Two diagonal drafts, as *a b*, *c d* (fig. 49), are run across the surface and connected by cross drafts, as *a d*, *c b*. The super-

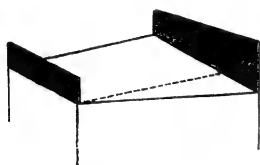
fluous stone is then knocked off between the drafts, until the surface coincides in every part with a straight-edge. 2nd, when the surface is small. In this case a chisel-draft is sunk along one edge of the stone, and a rule with parallel edges placed upon it. The workman then takes a second similar rule, and sinks it in a draft on the opposite edge, until the upper edges of the rules are out of winding, when the two drafts will be in the same plane, and the face may be dressed between the drafts.

137. *To form a winding Surface.*—For this purpose the workman prepares two rules, one with parallel, the other with divergent, edges; the amount of divergence depending on the distance at which they are to be placed

apart. These rules are sunk into drafts across the ends of the stone, until their upper edges are out of winding. The extremities of the drafts are connected by additional drafts along the sides of the block, the surface of which is then knocked off until it coincides throughout with a straight-edge applied in a direction parallel to that of the drafts.

The diverging rule is called the *winding-strip*, and the rules are called *twisting-rules*. The parallel rule will of course form a rectangle, whilst the form of the diverging rule will be that of a triangle with a rectangle

Fig. 50.



added to it. See fig. 50. As the width of the rectangular portion of the rules has nothing to do with the twist, we shall, throughout the following pages, consider the parallel rule as a straight line, and the winding-strip as a triangle, which will much simplify the diagrams.

In building oblique bridges with spiral courses, the latter are worked so that their winding-beds form portions of spiral planes; and the accurate determination of the *twist* is a problem of great importance.

138. We have already (articles 122 and 124, and fig. 43, plate 5) described the manner of tracing a spiral line on the surface of a cylinder.

If a cylinder be cut along a spiral line traced upon its surface in such a manner that the resulting section will everywhere coincide with a straight-edge applied perpendicularly to the axis of the cylinder, the surfaces thus produced are called spiral planes. A familiar example of a spiral plane whose width is equal to the radius of the circumscribing cylinder, is afforded by the

soffit of a cork-screw staircase, such as may be seen in many church towers.

139. *To find the Dimensions of the Winding-Strip for working a spiral Plane.*—In order that the principle on which the dimensions of the winding-strip are formed may be more clearly understood, we shall first assume the width of the spiral surface to be equal to the radius of the cylinder.

Fig. 51 is the perspective view of a quarter of a cylinder, of which fig. 52 is the development, and fig. 53 the right section.

Fig. 51.

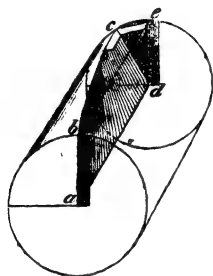


Fig. 52.

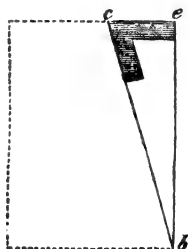
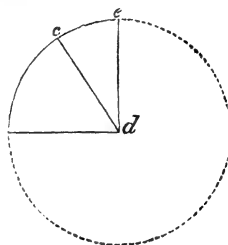


Fig. 53.



Let bc , fig. 52, be the development of the spiral bc , fig. 51. In fig. 53, make the arc $ec = ec$ in fig. 52; join dc , and the sector dec will represent the winding strip.

In applying the twisting-rules to the stone, they must be kept in parallel planes at a distance $= ad$, and perpendicular to the axis of the cylinder. It will be observed that the working edges of the rules will diverge from each other, the distance bc being greater than ad . To keep these edges, therefore, at the proper degree of divergence, it is convenient to connect the rules with light iron rods, of which the lengths can be readily

obtained from the development. If any difficulty is experienced in keeping the side of the winding-strip in a direction perpendicular to the axis of the cylinder, a small bevel may be used as shown in fig. 51, set to the angle ecb in fig. 52.

The twisting-rules should be made as thin as possible, and the working edges should be rounded, so that they may rest on the stone in the middle of their thickness only, as it would otherwise be necessary to form them to a winding surface.

The drafts ab , dc , fig. 51, having been sunk to the proper twist, the surface $abcd$ will be dressed off so as to coincide everywhere with a straight-edge applied to the two drafts with its ends equidistant from the points a and d .

140. If a straight line be drawn between any two points in the circumference of a spiral plane, it will not coincide with the spiral surface, and will only meet the latter in the extreme points lying in the circumference, and at a point midway between them. It should be clearly understood, therefore, that the process just described does not produce a spiral surface, although the approximation is so near in ordinary cases that the difference is scarcely appreciable, the distance between the twisting-rules being made so small, that for practical purposes the spiral bc may be considered as a straight line.

141. Let us now take the case of a spiral surface, whose width is less than the radius of the circumscribing cylinder.

Let figs. 54, 55, and 56, be respectively the perspective view, the development, and the right section of the quarter cylinder, the axial length bf being the distance at which the twisting-rules are to be applied.

Fig. 54.

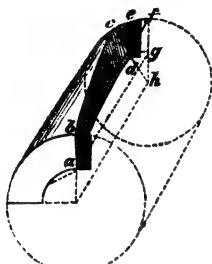


Fig. 55.

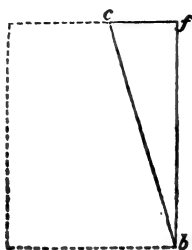
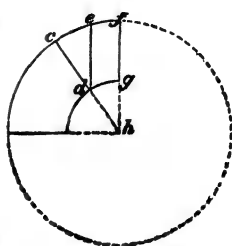


Fig. 56.



Let bc , fig. 55, be the development of the spiral bc , fig. 54: in fig. 56, make the arc $fc = fc$ in fig. 55.—Join ch , and from d , the point in which ch cuts the arch gd , draw de parallel to hf . Then dec represents the winding-strip. The mode of applying the twisting-rules is precisely the same as described in art. 139; in fact these rules are merely portions of the larger rules shown in fig. 51.

142. Instead of applying the twisting-rules *across* the ends of the stone as above described, some masons prefer placing them in the *length* of the bed. In this case the dimensions of the winding-strip are obtained on the assumption that it is a continuation of the *extradosal* cylindrical surface.

The working edges of the rules will be the same distance apart at each end, whilst their

Fig. 57.

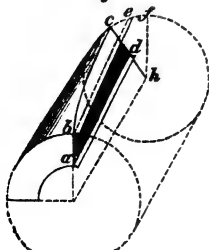
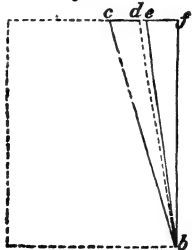


Fig. 58.



outer edges will be divergent.

Let figs. 56, 57, and 58 be respectively the right section, the perspective view, and the

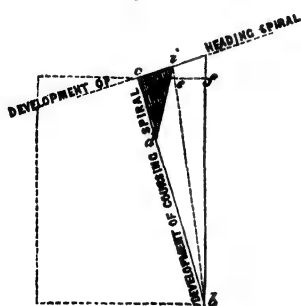
development, as before. Find $c e d$, fig. 56, as before. In the development, make $f e = f e$, fig. 56; join $b e$, then $b e c$ is the winding-strip.

We have already said that when the twisting-rules are to be applied to the *length* of the stone, the winding-strip is assumed to be a continuation of the extradosal cylindrical surface. But as the wide end of the winding-strip in reality is the chord of the arc $c e$, and as the working edges of the rules do not coincide with the extradosal and intradosal spirals, but are chords to them; the dimensions given by the process above described, would be subject to a slight correction, were they required to be mathematically correct. Practically, however, both the spirals and the arc $c e$ may be considered as straight lines, and the correction is therefore unnecessary.

The length of the working edge of the parallel rule will be found by setting off on the development $f d = g d$, fig. 56, and joining $d b$, which will be the length required.

143. There is yet a third way of obtaining the winding-strip, which is to consider it as a portion of a spiral heading plane. In the development, fig. 59, draw $c i$ perpendicular to $c b$, and meeting $b e$ produced in i ; set

Fig. 59.



off $c d = c d$, fig. 56, and join $i d$, then $d i c$ represents the winding-strip. This, again, is only an approximation, as the top of the winding-strip should not be a straight line, but a spiral, of which $c i$ is the development. This correction, however, is too trifling to be worth notice.

The working edges of

these twisting-rules will be applied with the same degree of divergence as those described in art. 141, but their outer edges will also be divergent, not parallel. The top of the winding-strip will form a right angle with the extradossal spiral.

144. Of the three methods above described, the first is the most accurate, as the dimensions of the winding strip are obtained correctly, whilst in the other two, the dimensions obtained are merely approximations, to which corrections must be applied if very great accuracy be required. The last method is, however, most convenient for the workman, who will always, unless otherwise directed, apply the winding-strip so that its wide end shall be square to the surface of the stone.

SOLID ANGLES.

145. Solid angles are those formed by the meeting of three or more faces in one point, and require for their execution two kinds of bevels, viz.:—

1. The face bevel, containing the angle formed by the meeting of two arrises bounding one of the faces.

2. The dihedral bevel, containing the angle formed by the intersection of two adjacent faces.

146. The angles of the faces, or as we shall term them, the *plane* angles, are best worked from a thin templet applied on the face of the stone, as shewn at *u B v* (fig. 60).

In making a bevel to work a *dihedral*, the sides of the bevel are set to the angle that would be formed by the intersection of a plane perpendicular to the common arris; and in applying the bevel to the stone, it must on each face be kept square to this line, as

shown at $r B j$ (fig. 60), making $A B r$, $A B j$, each right angles.

147. The solid angle occurring most frequently in practice, is that formed by the junction of *three* plane faces, to which the name of trihedral has been given. A trihedral has three plane angles and three dihedrals, of which six, any three being given the remaining three are also given, and may be obtained, all of them, by calculation, some of them by construction. We shall here, however, consider only how in those cases which are of most common occurrence they may be obtained by construction, viz.:—

1st. When the three plane angles are given.

2nd. When two plane angles and the included dihedral are given.

3rd. When one plane angle and the two adjacent dihedrals are given.

148. In each of these cases the remaining angles can be found by a simple geometrical construction; and as the lines to be drawn are the same in each case, it will save repetition to describe the whole of the figure in the first instance.

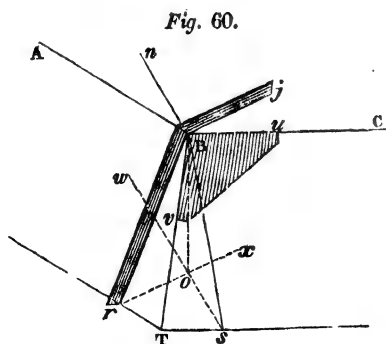
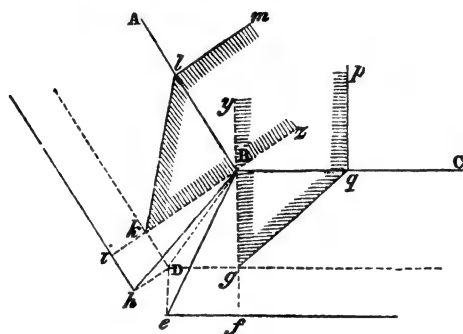


Fig. 60 is a perspective view of a trihedral, of which the faces $A B T$, $C B T$, are supposed to be bounded by a plane, $r T s$, parallel to the face $A B C$, at a distance, $B o$, measured at right angles to the face, $A B C$. The di-

hedral angles, $r B j$, $s B n$, adjacent to the face, $A B C$, are shown as formed by the intersections of the cutting planes, $x r B j$, $w s B n$, perpendicular respectively to the arrises $A B$, $B C$.

Figs. 61 and 62 are developments of the trihedral

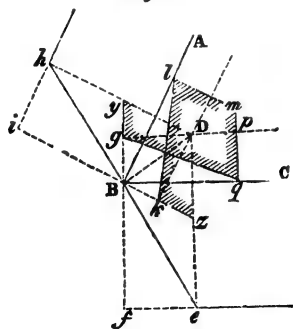
Fig. 61.



(the plane angles being in the one case all obtuse, in the other all acute), the plane angles $h B A$, $A B C$, $C B e$, corresponding to the angles $T B A$, $A B C$, $C B T$, in fig.

60, and the lines $B h$, $B e$, being of equal length and corresponding to the arris $B T$ (fig. 60), $h i$, $e f$ being

Fig. 62.



parallel respectively to $A B$, $B C$. Erect the perpendiculars $B i$, $B f$; draw $e D$, $h D$ respectively, parallel to $f B$, $i B$, and meeting each other at D ; join $B D$, and draw $D k$, $D g$ respectively parallel to $A B$, $B C$. Then $A B D C$ will be a plan on a plane of projection parallel to the face $A B C$, $B g$ being

$= o s$ (fig. 60), and $B k = o r$ (fig. 60). Now, in order to project the sections $x r B j$ and $w s B n$ (fig. 60), set off on the lines $B A$, $B C$ (figs. 61 and 62), equal

distances $B l$, $B q$, corresponding to the perpendicular distance $B o$ (fig. 60) of the two parallel planes; erect the perpendiculars $l m$, $q p$; and join $l k$, $q g$. If $i B$, $f B$ be produced to any points, $z y$, beyond $k g$, respectively, then $z k l m$, $y g q p$ will be the respective projections of the sections $x r B j$, $w s B n$, so that $m l k$, $p q g$ are equal to $j B r$, $n B s$; and $l k$, $q g$ to $B r$, $B s$, that is, to $B i$ and $B f$ respectively.

149. In applying this diagram to practice, $A B C$ is always made one of the given angles, and the perpendiculars, $B i$, $B f$, having been drawn of convenient lengths, the remainder of the figure is completed either from the plane or the dihedral angles, as the case may require.

150. Case 1. *Given three plane Angles of a Trihedral to find the Dihedrals.*—Draw the development and find the point g , as in art. 148. With g as a centre, and radius $g q = B f$, describe an arc cutting $B c$ at q . Join $g q$, and draw $q p$ perpendicular to $B c$; then $g q p$ will be one of the dihedrals, and the other two may be found in a similar manner.

151. Case 2. *Given two plane Angles and their included Dihedral to find the remaining Angles.*—Let $A B C$, $e B C$, be the given plane angles, and $g q p$ their included dihedral angle. Having found the point D , draw $D h$ parallel to $B i$, and with B as a centre and radius $B e$, describe an arc cutting $D h$ at h : join $B h$, and $A B h$ will be the remaining plane angle. The remaining dihedrals will be found as in art. 150.

152. Case 3. *Given one plane Angle and two adjacent Dihedrals to find the remaining Angles.*—Let $A B C$ be the given plane angle, and $k l m$, and $g q p$ the adjacent dihedrals. Make $B i$, $B f$ respectively

equal to lk , qg . Draw dh , de , ih , fe as described in art. 148, and join bh , be : then abh , cbe are the remaining plane angles, and the remaining dihedral can be found as in art. 150.

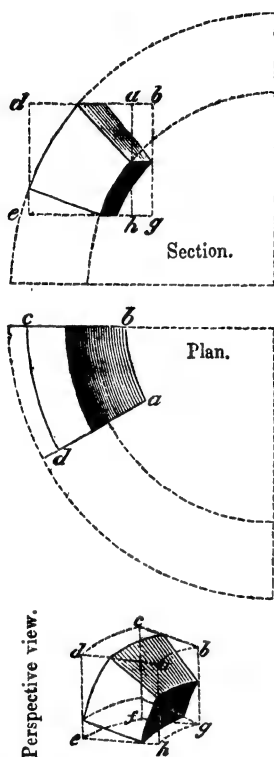
SURFACES OF OPERATION.

153. No difficulty occurs in working a block of stone, of which the faces, beds and joints, are to be either vertical or horizontal planes, as the several dimensions required can be obtained directly from the plan and elevation. Nor is any difficulty introduced if some of the surfaces are cylindrical, as a cylindrical surface can be worked with almost as much facility as a plane; the only difference being, that a curved rule is used in one direction and a straight one in the opposite, whilst in the latter case the straight-edge alone is used.

154. If, however, any of the sides of the block are to be formed into conical, spherical, or spiral surfaces, the matter becomes somewhat complicated, and it is necessary first to bring the stone to a series of plane or cylindrical surfaces on which to apply the bevels and templets required for finishing the work. These preparatory surfaces are called *surfaces of operation*. In cases where the blocks are of large size, they are brought to their approximate shape at the quarry, and it is of importance that the quarryman should be enabled to do this in such a manner as to reduce the subsequent labour of the mason as much as possible.

155. The simplest plan is to make the surfaces of operation either horizontal or vertical, by which means the lines required for making the bevels and templets

Fig. 63.



can be taken directly from the plan and section which are horizontal and vertical projections. Thus, let it be required to work a voussoir of a dome—we may first work the block roughly, so as to form a portion of an upright hollow cylinder, as shown by the dotted lines in fig. 63, and transferring the lines of the plan and section to the surfaces of operation thus formed, the subsequent operations become very simple.

When the stones are small, and stone abundant, this will generally be the best mode of proceeding; but, with large blocks, the waste of material and labour would be very serious, and it is necessary to use such methods as will enable us to economise the material as much as possible.

PART II.—APPLICATION OF PRINCIPLES TO PARTICULAR CONSTRUCTIONS.

156. Having now explained the general principles of stonecutting, we proceed to show their application to some few particular constructions, each of which may be regarded as the type of a class to which the same

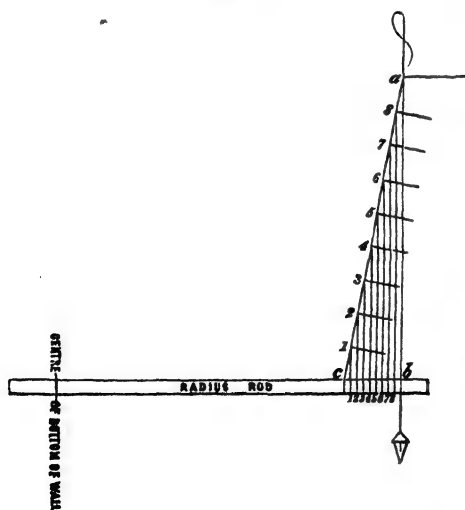
rules are applicable, with such trifling modifications as the circumstances of each individual case may render desirable.

157. *Curved Wing-Walls.*—To execute a wall with a straight batter on a curved plan requires much care and attention, and a considerable number of templets for the proper working of the conical beds of the courses, and for obtaining the twist of the coping.

We have already described in detail the manner of constructing the several projections required in designing a conical wall, and therefore need not say anything further on the subject in this place, but will proceed at once to describe the manner of obtaining the necessary templets, and of working the stone.

158. *Arrangement of the Courses.*—On a platform

Fig. 64.



draw a straight line equal to the vertical height of the wall at its highest point, calculate how much the wall will batter in this height, and set off the distance at right angles to the first line, as shown in fig. 64, where ab is the vertical

height of the wall, and bc the amount of batter. Draw

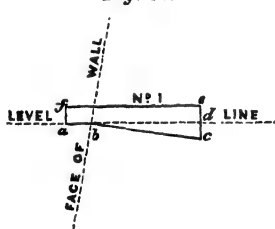
the face line ac , and divide it into the intended number of courses.

When the stone provided for the work runs of various thicknesses, measure the thickest and the thinnest blocks and gauge the bottom and top courses accordingly; set off these dimensions on the face line ac , and arrange the intermediate courses as described in art. 78, section II. Number the bed-joints as shown in the figure, beginning from the bottom of the wall.

Provide a rod, and mark on it the radius of each bed-joint, numbering each joint in succession to correspond with the numbers on the line ac .

159. *To work the top Bed**.—The beds of the courses of a battering wall are made to dip at right angles to the face, whilst their front arrises lie in horizontal planes. The first operation, therefore, will be to form a horizontal surface of operation on which to apply a curved templet, cut to the radius of the front arris.

Fig. 65.



Make a bevel, as shown in fig. 65, so that the angle abc shall be the dip of the bed. The length of bc will be regulated by the width of the stones to be worked; that of ab by their length—the width, de , of the rectangular portion, $adef$,

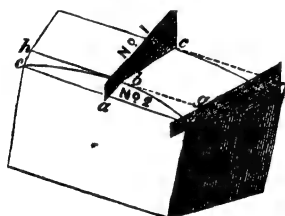
* We are not aware that this method has been previously published. The method most commonly in use is the first of the two methods described by Mr. Peter Nicholson in his "Practical Masonry," &c. Mr. Nicholson's rule is a very excellent one, but the construction of the hyperbolic templet for obtaining the wind of the bed is too complicated to be understood by an ordinary workman. In the rule here given, the lines of the templets are those of the work itself, and can be taken directly from the plan and section.

is of little consequence; 3 inches is a convenient dimension. Call this bevel No. 1. It will apply to the whole of the courses.

Make a curved templet to the radius of the front arris, as set out on the rod described in art. 157. The length of this templet must be a little more than that of the longest stone in the course. Call this templet No. 2. Each bed-joint requires a separate templet, but the same templet will work the *top* bed of one course and the *bottom* bed of that next above it.

With No. 1 sink a draft abc (fig. 66) across the

Fig. 66.



centre of the length of the block, so that ab is equal to the versed sine of the curve of the front arris. Through b draw hbg , perpendicular to bc , and knock off the front edge of the block, so as to form the horizontal

surface of operation $ea f g b h$. On this surface apply No. 2, and draw the curve of the front arris $e b f$, keeping the curve perpendicular to bc . Make a duplicate of No. 1, and with these two rules bring the top bed to its proper wind. To do this, one rule must be placed at bc , and the other on successive portions of the surface, the rule being kept square to the curve line, $e b f$, and placed so that the point b coincides with it. The second rule must then be sunk till the upper edges of both rules are out of winding. (See fig. 66.)

On the bed thus worked draw a line square to the front arris as shown in fig. 67, and make a flexible templet to the angle $e b c$. This templet when laid flat will be a portion of the development of the conical surface of the bed, and when bent round the stone will give

Fig. 67.

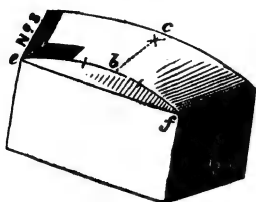
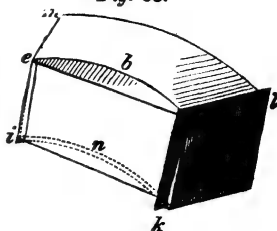


Fig. 68.



the direction of the joint. Call this templet No. 3. Each course requires two templets, but the same templet will work the *top* bed of one course and the *bottom* bed of that next above it. Gauge the top bed to a regular width, and mark off the radiating ends with No. 3. The stone is now brought to the state shown in fig. 68.

160. *To work the Face.*—With a common square applied at the ends of the top bed, sink a draft at each extremity of the face as shown in fig. 68. On these drafts mark the thickness of the course as shown at ik . Take No. 2, corresponding to the front arris of the bottom bed, and sink the draft $in k$ —keeping the templet so that $bn = ei$, the thickness of the course. Work the face between the top and bottom drafts with a straight-edge. Gauge the arris line $in k$ parallel to ebf . Draw a line bn (fig. 69) square to the top arris, and make a flexible templet to the angle ebn . This templet when laid flat will be a portion of the development of the conical face of the wall, and, when bent into the curved face, will give the direction of the upright joint in the face. Call this templet No. 4. A separate templet will be required for each course. Complete the working of the face by marking off the face joints ei, fk with No. 4, as shown in fig. 69.

161. *To work the Ends.*—The ends of the stone will

be vertical planes, and are therefore worked with a straight-edge applied to the arris lines lf , fk and me , ei , fig. 69.

Fig. 69.

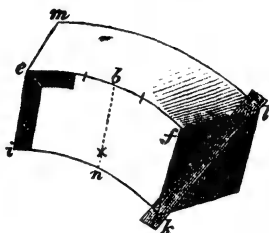
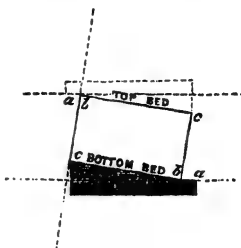


Fig. 70.



162. *To work the bottom Bed.*—This is done with No. 1 and its duplicate, simply reversing the rules, end for end, as shown in fig. 70, keeping the point c on the arris ink . The top bed is round, and is worked from the centre to the ends. The bottom bed is hollow and is worked from the ends to the centre.

163. *To build the Wall.*—Set up an iron rod at the centre of the cone, and steady it as may be most convenient (see fig. 71, plate 7; in which, however, the stays are omitted to avoid confusing the drawing).

Provide two battering rules, on which mark the bed-joints and fix them very accurately at the extremities of the wall. Then, as each course is laid, try its correctness with the rod described in art. 158, or with a stout measuring tape, of which the ring is passed round the iron rod at the centre of the cone.

These precautions are especially necessary in building curved walls either in brick or in rubble, as without being able to refer to a centre it is very difficult to keep the courses to the proper curve. In building an ashlar wall, the stones being previously brought to the curvature of the face, this difficulty is much lessened; but the ap-

pearance of the coping depends so completely on the accuracy with which the work is carried up, that the use of the centre rod cannot well be dispensed with without a risk of the coping being slightly crippled.

164. *To form the Top of the Wall to receive the Coping.*—Shift the rod to n , fig. 72. String two lines in the plane of the front edge of the top of the wall, as shown at nb, nb , figs. 71 and 72, plate 7. On a platform strike out a quarter ellipse, with semi-axis major $= nb$, and semi-axis minor $= qn$. Make a templet to this curve, in any convenient number of pieces, and call this templet No. 5. Beginning at q , fig. 72, place No. 5 against the face of the wall, piece by piece, keeping it out of winding with nb, nb , and draw the line of the front arris on the top stones, which will have been carried a little above this line. The top of the wall must then be dressed off to this line, keeping the surface level in the direction of the centre rod, and it will then be ready to receive the coping.

165. *To work the Coping.*—Divide the front edge of the wall into the number of stones which the coping is to contain, and square the joints across from the face. Let it be required to work the stone No. 3, fig. 72—from a point b in the front of the wall, corresponding to the centre of the stone, draw two lines perpendicular to the arris, viz., ab on No. 5, and bc on the top of the wall. Make a bevel to this angle, and call it No. 6. A separate bevel will be required for each coping-stone.

In working the stone it is convenient to begin with the top surface. The block being roughly scappled to its shape; with No 6 sink a draft, abc , across the centre of the top surface, as shown in fig. 73, plate 7, and form a surface of operation, $efgbh$, as described in art. 159; the only difference being that, in the present

case, the surface of operation is not horizontal, but lies in the plane qnb , fig. 72. On this surface apply No. 5, by which draw the front edge, ebf . The dimensions of the twisting-rules for working the top surface are found by actual measurement from the wall itself. The number of twisting-rules required for each stone will vary according to the degree of twist, which increases from the foot of the wall upwards*. These twisting-rules must be applied in a direction radiating from the face line, and the workman must commence at the centre of the stone, on the draft bc , and work each way to the ends.

The top surface having been worked to the proper twist, the radiating joint lines are marked with a bevel; the angles being taken from the lines previously marked on the top of the wall.

The width of the top of the stone is then gauged parallel with the front line, and the fronts and backs worked. The ends should be left rough until the whole of the coping is worked, in order to ensure an accurate fit.

In applying the square to work the front face, it should be placed so that a plumb-line suspended from any part of the front edge shall coincide with the face. If the square be applied *perpendicular* to the arris, the bottom edge of the coping will appear underset, which has a wretched effect†.

The front and back having been worked, the stone is

* It must be remembered, that the top and bottom arrises of the coping do not lie in parallel planes, as explained in art. 117. The difference in the twist of the top and bottom beds arising from this cause is scarcely appreciable, but it may be allowed for in taking the twist from the top of the wall, when the curve is sufficiently sharp to make this necessary.

† Some persons prefer to make the front of the coping, battering; this is simply a matter of taste.

gauged to its proper thickness on each side, and the bottom bed can be worked with a straight-edge applied between these lines.

166. We have described the front of the coping to be worked with No. 5, and this would be perfectly correct were the coping set flush with the face of the wall. But, as it is always overset to a slight extent, say two inches, it is evident that the front of the coping must be made to curve a little quicker than the front of the wall. In ordinary cases, the difference between these two curves is not perceptible in the length of a stone, but when the radius of curvature is very small, two templets will be required, one to work the top of the wall, and the other to work the front of the coping.

167. There are very many ways in which the twisted coping of a wing-wall may be worked; but the foregoing method appears to us the simplest, and that requiring fewest templets. As each stone requires a separate bevel, face-mould, and twisting-rules, the trouble of making the templets adds greatly to the cost of working the stone, and the last-named consideration is therefore one of great importance.

168. Walls are sometimes built on elliptical plans, but this should always be avoided if possible, as the working of the stone is a very complicated process.

DOMES.

169. The foregoing rules apply with trifling modifications to the execution of domes, spherical niches, and fan vaults; and generally to all constructions in which the beds of the courses form conical surfaces, and of which the joints lie in vertical planes. A single example will suffice to show the nature of these modifications.

Let it be required to work the voussoir *abcdefg*, fig. 74, plate 7, of a hemispherical dome. The top bed in this case is hollow, and the bottom bed rounded, therefore begin with the latter. Obtain the front arris, and work the bed as in art. 159. Work the face, *adef*, so as to form a conical surface of operation, as in art. 160; except that the chisel drafts at the ends of the face will be sunk with a *bevel* set to the angle *adc*, instead of a common square being used. Work the ends as described in art. 161. Obtain the arris *af*, and work the top bed as described in art. 162; except that, as the inclinations of the top and bottom beds are not the same, there must be as many sets of winding rules as there are courses. Lastly, work the conical surface of operation to the proper curve, with a curved rule, as shown in the figure.

ARCHES.

170. The construction of either circular or elliptical arches, of which the abutments are square with the face, offers but little difficulty.

As the depth of the arch-stones is generally greater than their thickness, the workman commences by working one of the beds. This being done, the ends are squared, and their exact shape marked from a templet. The opposite bed is now worked to the lines thus found. Lastly, drafts are sunk at each end of the soffit to the curves previously marked, and the soffit is dressed off to coincide with a straight-edge applied between the drafts in the length of the stone. This will be understood by inspection of fig. 75, plate 7.

Another method is to work the soffit from the bed first formed, by means of an arch-square or curved bevel, as shown in fig. 76, plate 7. One bed and the

soffit being worked, the other bed is worked from the soffit in the same way. This method dispenses with the necessity of squaring the ends before working the soffit, which is sometimes an advantage.

In both these methods the straight-edge is used for trying the surface between the chisel-drafts at the ends of the curved soffit; but in the first method these drafts are got by applying a templet on the squared ends, and in the second by means of the arch-square.

171. *Oblique Arches*.—We have in the first section of this volume described the different methods in which oblique arches may be constructed. In the following pages, therefore, we propose to speak only of cylindrical arches built in spiral courses, of which the beds radiate from the axis of the cylinder. The reader who will take the trouble thoroughly to master the rules here laid down, will find no difficulty in executing any other description of arch.

The construction of a skew arch of which the span, rise, width of soffit, and angle of skew are given, requires but very few lines to be drawn for finding the templates and bevels. But it is always desirable, before commencing operations, to make a large drawing to an inch scale, for the purpose of ascertaining the sizes of the stones that will be required, the best manner of arranging the heading joints in the soffit, and such other particulars as cannot well be obtained from a small sketch; and we shall, therefore, briefly describe the projections and developments that are required for this purpose. (See fig. 77, plate 7.)

172. *Plan*.—Draw two lines, cb, dy , parallel to each other, at a distance, de , corresponding to the square width of the soffit of the arch. Set off the angle of the bridge, and draw one impost line, cd . Draw the

second impost line parallel to the first, at a distance, $a b$, corresponding to the span of the arch on the square section.

173. *Section*.—With the given span $a b$, and rise $h i$, draw the square section of the arch.

174. *Development*.—Draw a development, $b g k y$, of the soffit of the arch from the data thus obtained. Draw on it the development of a heading spiral, passing through the extremities of the impost lines in one of the fronts. Divide this line into any convenient number of equal parts, as 13, corresponding with the intended number of stones in each face of the arch; an uneven number being always taken to allow for a key-stone.

From k , the opposite end of the impost line making an acute angle with the face, let fall a perpendicular, $k l$, on the heading spiral just drawn, which will represent the development of a coursing joint. If this line pass through one of the divisions on the heading spiral, the design may be proceeded with without any alteration of the dimensions; but this will most probably not be the case. It will then be necessary to adjust the dimensions, so as to make the coursing spiral pass through one of the divisions; which may be done—

1st. By altering the width of the bridge.

2nd. By altering its span.

3rd. By altering the angle of skew; or, lastly, by a slight adjustment applied to all these data.

If the dimensions of the arch are unalterably fixed, this first coursing joint must be drawn through the nearest face-joint; but, in this case, as the coursing and heading spirals are not perpendicular to each other, the soffits of the stones will be out of square*, which is very objectionable. In building arches of brick, with stone

* That is, if the stones are worked to form regular bond,

quoins, as shown in fig. 78, plate 8, this difficulty is scarcely felt, because it is not necessary that the face-joints of the opposite fronts should range with each other; all that is required being that they should coincide with some joint of the brickwork, so that, in this case, the necessary adjustment can never exceed half the thickness of a brick.

The angle made in the development, by the intersection of the coursing joints with the impost, is called the angle of intrados. The corresponding angle, in a development of the extrados, is called the angle of extrados.

175. *Arrangement of Heading-Joints.*—Divide each impost into as many parts as there are divisions cut off on the heading spiral by the coursing joint first drawn, which, in this example, are five in number; and through the divisions on the impost, and on the heading spiral, draw the developments of the coursing spirals, which will be parallel to and equidistant from each other. Through the divisions on the imposts draw heading spirals parallel to that first drawn, and arrange the heading joints on these lines and on others parallel to them, so as to form regular bond throughout the whole of the soffit. (See fig. 11, plate 2, art. 39.)

It will be seen that, from the heading spirals not being parallel to the face line, the quoin-stones will be of very irregular lengths; and this is particularly conspicuous in brick arches with stone quoins, whose ends are portions of continuous heading spirals. The best way of avoiding this is to draw on the development lines parallel to the face lines at distances corresponding to the intended lengths of the long and short quoins, and to make the end of each quoin a portion of a separate heading spiral, passing through the intersections

of these lines with the coursing joints, as shown in fig. 78, plate 8. Some persons, in building brick arches with stone quoins, make the ends of the latter parallel with the face line, which is very objectionable, as it throws them out of square with the brickwork, which is offensive to the eye, and makes unsound work.

176. *Skewbacks*.—The next thing to be considered is the arrangement of the joints in the imposts. The top of each impost must be cut into checks or skewbacks to receive the ends of the courses; and, as the beds of the courses are worked to radiate from the centre of the cylinder, the checks will be square to its axis, and to the faces of the abutments, as shown in fig. 77. In settling the sizes of the stones forming the imposts, it must be borne in mind that the stone at the obtuse quoin will be wider at the back than at the front, whilst the reverse takes place at the acute quoin; and it is of importance that the latter stones shall be of sufficient size to bond into the rest of the work. The thrust of a properly built skew arch being in a direction parallel to its fronts and not at right angles to the abutment, it will always be desirable to make the joints of the masonry square to the fronts, and, therefore, the backs of the impost stones should be cut so as to bond with the rest of the masonry, as shown in fig. 79, plate 8.

The last thing to be attended to in the design is the elevation of the arch, and the arrangement of the courses of the spandrels.

177. *To draw the Elevation*.—The curves of the intrados and of the extrados are both portions of ellipses, of which the spans are to be taken from the plan and the heights from the section. The positions of the joints on the intrados are taken from the divisions on the *face line* of the development of the intrados. Their position on

the extrados may be formed by developing the extrados, the manner of doing which may require some little explanation.

Since the joints are made to radiate in a direction perpendicular to the axis of the cylinder, it follows that the axial lengths* of the intradosal and extradosal spirals will be the same, see fig. 77, plate 7; but, as the circumference of the extrados is longer than that of the intrados, the angles made with the abutments by the extradosal spirals will be greater than those made by the coursing joints in the soffit; or, in other words, the angle of extrados will be greater than the angle of intrados, and, as a consequence, the extradosal plans of the stones will be out of square, as shown in the development of extrados, fig. 77. In drawing the heading spirals in the development of the extrados, they will not be perpendicular to the coursing spirals, nor will they pass through the intersections of the face and impost lines, but they will fall within the face at the obtuse, and beyond it at the acute, quoins. This will be fully understood by reference to the figure. Divide the extreme heading spirals into as many equal parts as the heading spirals of the intrados, and through these divisions draw the developments of the extradosal coursing joints. Transfer the divisions on the *face* line of the extrados to the curve of the extrados in elevation, and draw in the face-joints, between the points thus determined in the extrados and intrados. In strictness, the face-joints are not straight lines, but curves; as every intersection of a plane with a spiral surface will be a curved line, except when the plane of intersection is perpendicular to the axis of the cylinder; but, unless

* By axial length is here meant the distance measured on the axis of a cylinder, corresponding to an entire revolution of a spiral.

the bridge is very much on the skew, the curvature is not worth noticing in the drawings, as its omission does not in any way affect the work.

178. *Focal Eccentricity*.—It was first pointed out by Mr. Buck*, that the face-joints of an oblique arch of equal thickness from the springing to the crown have a remarkable property; viz., that their chords all radiate from a point below the axis of the cylinder, the distance increasing with the angle of obliquity; and he gives in his work the following simple rule for ascertaining this distance, which he calls the focal eccentricity, see the lower part of fig. 77, plate 7. Draw ab = radius of extrados, and bc perpendicular to it, making the angle acb = angle of skew; draw cd perpendicular to bc , making the angle cbd = angle of intrados; then cd is the focal eccentricity. For the demonstration of this rule we refer the reader to Mr. Buck's work. By taking advantage of this property of the face-joints, we can draw the elevation without the trouble of making a development of the extrados, which saves much time.

179. *Spandrels*.—The face-joints having been drawn, the last thing to be done is to arrange the lengths of the quoin stones, and the heights of the spandril courses. This is sometimes troublesome to manage, as it is necessary for the appearance of the work that the heights of the spandril courses should diminish regularly from the springing to the crown, and the lengths of the quoins must be adjusted so as to effect this. If the elevation is carefully drawn to an inch scale, the lengths of the quoins can be obtained from it with sufficient accuracy without drawing a full-sized elevation; which is an expensive operation, as it requires a large extent of platform.

* "A Practical and Theoretical Essay on Oblique Bridges," by George Watson Buck, M.I.C.E.—Weale, London, 1839. (See chap. II.)

We now proceed to describe the manner of finding the bevels and templets for the execution of the work.

180. In fig. 80, plate 8, let

$\angle d c b$ = angle of skew.

$a b$ = span on square.

$h i$ = versed sine of arch.

$d e$ = width of soffit on the square.

$a x$ = radius of intrados.

$c b$ = oblique span.

$c d$ = length of impost.

$b f$ = development of square section.

$b g$ = development of heading spiral.

$\angle g k l$ = angle of intrados.

Of these data, the four first are known, and the others must be found from them, either by geometrical construction on a platform, or by calculation; the latter plan being the most correct, the quickest, and the cheapest.

181. *Radius.* $a x = \frac{a h^2 + i h^2}{2 i h}$ see art. 83.

182. *Oblique Span.* $c b = a b \times \operatorname{cosec} \angle d c b$.

183. *Length of Impost.* $c d = d e \times \operatorname{cosec} \angle d c b$.

184. *Development of Square Section.*—The natural sine of the angle $a x h = \frac{a h}{a x}$ Look in a table of

natural sines for this number, and call the corresponding number of degrees, n ; then, $b f = a i b = 2 n \times a x \times .017453$ (as in art. 88).

185. *Development of Heading Spiral.* First $f g = a c = a b \times \cotang \angle d c b$.

Then $b g = \sqrt{b f^2 + f g^2}$ = development of heading spiral.

186. *Angle of Intrados.* $\frac{f g}{b f} = \text{sine of } \angle f b g = \text{sine of } \angle g k l$.

187. The above calculations are best performed by the aid of a table of natural sines, secants, and tangents, without using logarithms, and may be made with a table of natural sines only, but the operation is somewhat tedious in the latter case, as it involves dividing by the sine of the angle of skew, which is very troublesome, as the sine should not be taken to less than six places of decimals—

$$\text{Thus, the oblique span, } b = \frac{a b}{\sin \angle d c b}.$$

$$\text{and the impost length } d c = \frac{e d}{\sin \angle d c b}.$$

188. These dimensions having been calculated, the lengths of the imposts and of the development of the heading spiral must be set out very exactly on long rods, and divided into the number of equal divisions previously determined on. The divisions on the impost rod will give the exact length of the checks to be cut on the springers, and the divisions on the other rod will show the exact width of the courses.

189. *Templets for the Skew-backs.*—On a sheet of zinc, draw two lines at right angles to each other, as $a b, b c$, fig. 81, plate 8; set off $b c =$ width of a course on the soffit, and $b a = b c \times \cot \angle$ of intrados. Join $a c$. Then the triangle $a b c$ will be the form of the templet for the impost checks in the soffit, and $a c$ should exactly agree with the length of check previously ascertained. From b let fall on $a c$ the perpendicular $b d$. On a platform draw a straight line $a b c$, fig. 82, plate 8, making $a b =$ radius of intrados, and $b c =$ thickness of arch at springing. With centre a , and radius $a b$, draw the arc $b e = b d$, fig. 81. Through e draw $a e d$ making $e d = b c$. With centre a , and radius $a c$, draw $c d$,

cutting aed in d . In fig. 81, produce bd to e , making $dbe = dc$ in fig. 82; join ae, ec ; then aec is the form of the templet for the impost checks on the extrados.

190. In working the springers, they are first brought into a cylindrical form, and divided into the proper number of checks by the impost rod. The templets are then applied on the intrados and extrados, and their profiles marked on the stone, which is then cut away to these lines.

191. *Twisting-Rules.* On the platform set out the angle of intrados, as gkl , fig. 80, plate 8, and let km be the axial distance at which the parallel ends of the twisting-rules are to be applied. Draw mn perpendicular to kg ; nk will be the distance between the rules on the intrados. On the platform with radius of intrados, mx , fig. 83, plate 8, draw $nm = nm$ in fig. 80. Draw xmp and xno , and the concentric arc op , making $on = mp =$ thickness of arch. On nm produced, fig. 80, set off $mo = po$, fig. 83. Join ko , then ko is the distance at which the twisting-rules are to be applied on the extrados. In fig. 83, draw nq parallel to mp ; then noq will be the divergent portion of the winding-strip.

When a bridge skews to the *left*, as shown in fig. 80, plate 8, the winding-strip must be applied on the *right-hand side* of the parallel rule, and *vice versâ*.

The rules here described are to be applied as directed in art. 141.

192. *Templet for the Curve of the Soffit* *.—The por-

* The reader must not be discouraged if he do not, on the first perusal, understand the object of the operations here described. Some assistance may be derived from an inspection of fig. 77, in which figs. 84, 85, 86, and 87, are repeated on a small scale in connection with each other; but the best plan would be to lay down the several lines on the surface of a cylinder, when the principle on which the rule is founded becomes immediately apparent.

tion of a coursing spiral included in the length of any voussoir may be treated as an arc of a circle, and may be obtained approximately as follows:—Draw on the platform the lines ab, bc , fig. 84, plate 8, of any convenient length, making $\angle abc =$ angle of intrados. On ab let fall the perpendicular ca . In fig. 85, plate 8, draw $ax =$ radius of intrados, and with x as a centre, draw the arc $ac = ac$, fig. 84. From c let fall on ax the perpendicular cd . Draw two lines parallel to each other, fig. 86, at a distance apart $= ad$, fig. 85. From any point, b , in one line as a centre, with radius $= bc$, fig. 84, describe two arcs cutting the other line, as shown at a and c . This will give three points in the curve, which may then be drawn in with a trammel, as described in art. 64. Make a templet to the curve thus found, and call it templet No. 1.

193. *Templet for marking the Heading-Joints on the Beds.*—Take a sheet of zinc, and mark on it the curve of the soffit with templet No. 1. With intersecting arcs, set up a perpendicular to the curve as ab , fig. 87, plate 8, making $ab =$ thickness of arch, or a little more. Cut the zinc to the angle bac , as shown by the shaded part of the figure, and this will be the templet required; which call No. 2.

194. *Templet for marking the Heading-Joints on the Soffit.*—This is simply a rectangular piece of zinc of any convenient length, and of which the width is that of a course. It is best, however, to make it the length of the longest voussoir. Call this templet No. 3; see fig. 89, plate 8.

195. *Arch-Square.*—The arch-square, required for working the soffit from the bed, is precisely similar to that shown in fig. 76, art. 170, and needs no further description.

196. *Method of working the Voussoirs.*—1st *Bed.*

Bring one side of the stone to a plane surface. With No. 1, draw on it the curve of the intradosal coursing joint, as *abc*, fig. 88, plate 8. With No. 2, draw one of the heading-joints, as *ae*. Take the twisting-rules, and, applying the parallel rule to the line just drawn, work the bed *abcde* to the proper twist. With No. 2, draw the second heading-joint *cd*. This completes one bed.—*Soffit*. With the arch-square applied so that it shall be always in a plane perpendicular to the axis of the cylinder, work the soffit to a cylindrical surface. If any difficulty is found in applying the arch-square in the proper direction, a small bevel may be applied to the soffit, set to the complement of the angle of intrados, as shown in fig. 89, plate 8. With No. 3 gauge the soffit to its proper width, and mark the heading-joints. This completes the soffit.—*2nd Bed*. This is worked from the soffit with the arch-square, and the heading-joints drawn with No. 2.—*Ends*. These are worked with a straight-edge applied between the joint-lines drawn on the beds with No. 2.

197. *Centering*.—As soon as the abutments have been carried up to the springing, and the impost stones set, the centering must be erected. The ribs should be placed parallel to the face, and not square to the abutments; as the former plan ensures greater accuracy in the curvature of the fronts. The laggings must be securely fastened, and their upper surface planed perfectly true, so as to coincide everywhere with a templet cut to the curve of the soffit. Too much importance cannot be attached to this, as upon it mainly depends the accuracy of the work.

The surface of the laggings having been made perfectly true, the lines of the coursing and heading-joints must be marked upon it, to assist the workmen in setting the arch-stones.

This is done in the following manner; which will be understood by reference to fig. 11, plate 2.

Draw the face lines, and, having bisected them, draw a level line along the crown of the centering from centre to centre of each face.

Take the impost rod, and transfer the divisions on it to this centre line. Prepare a thin flexible board as a straight-edge, and, having planed its edges very true, transfer to it with great care the divisions of the heading spiral, which must be set off from the rod previously prepared, as described in art. 188. This straight-edge need not be longer than is necessary to extend from the impost to the crown of the arch. Then, beginning at the extremities of one of the imposts, bend the straight-edge round the centering, and draw a series of heading spirals, from impost to impost, through the divisions on the centre line, and the corresponding lines of the checks on the springing stones. It may be necessary to observe that the laggings must project a little way beyond the fronts of the arch, or there will not be room for drawing the extreme heading spirals. Transfer the divisions on the straight-edge to these heading spirals, taking care that the centre line at the crown passes through the *centre* of a division in each case. Through the points thus found, draw the coursing spirals, which will exactly coincide with the coursing joints in the soffit of the arch. The heading-joints must then be marked, and the numbers of the arch-stones painted on, so that no delay shall occur in setting the stones, from their being brought in the wrong order.

198. *Face Quoins.*—*Templets for Soffits.*—The soffits of the ordinary voussoirs are rectangular; but this is not the case with the quoins, the soffits of which are all out of square more or less. The templets for marking off the face line on the soffits of these stones are best

obtained from the lines on the laggings, which is done by bending round templet No. 3, and cutting off the end to coincide with the face line.

199. *Templets for Angles of Coursing and Face-Joints.*—In the ordinary voussoirs the heading-joints are all perpendicular to the curve of the soffit. This is not the case with the face-joints, which make varying angles with the coursing joints, according to their distance from the springing; the joints lying between the crown and the acute quoins, making acute angles with the soffit joints, whilst the angles on the opposite halves of the fronts are obtuse angles. There are many ways of obtaining these angles by geometrical constructions; but these methods are very intricate, and require a great many lines. We prefer, therefore, to take these angles at once from the lines on the centering, which may be done with great facility and accuracy as follows:—

Apply templet No. 1 to a thin strip of deal; and, having marked on the latter the curve of the soffit, cut away the superfluous wood, so as to make a corresponding concave rule. Take this rule and frame it to three arch-squares, set in planes perpendicular to the axis of the cylinder, as shown in fig. 90, plate 8; so that when the curved edge, abc , is placed on a coursing joint on the centering, the curved edges of the arch-squares shall coincide with the surface of the laggings. Mark the centre of ac as shown at b . Then, beginning at the joint nearest to the acute quoin, place the edge abc to coincide with the coursing joint; and so that the face line shall pass through the point b . Take a plumb-line with a pointed bob, and pass it carefully along the arris def until the point of the bob is exactly over the face line. Mark this point as shown at e . Then abc will

be the angle required, and $e b c$ will be the angle for the corresponding obtuse quoin. Find the angles for the other joints in the same manner. Take templet No. 2, place it so that its curved side corresponds with $a b$, and cut the templet to the angle $a b e$, and this will be the templet for marking off the face-joint on the adjacent beds of the two first courses from the springing. The remaining templets will be constructed in the same manner.

200. *Angle of Twist*.—As in all books on skew masonry a great deal is said about the angle of twist, it may be desirable that we should say a few words on the subject. The term angle of twist is an expression used to denote the difference between the angles of intrados and extrados, and is often erroneously spoken of as synonymous with the *angle of the twisting-rule**. Thus in figures 57 and 58 (art. 142) the angle $c b e$ is the angle of the twisting-rule, and $c b d$ is the angle of twist, being a somewhat smaller angle, which must necessarily be the case, as may be seen by inspection of fig. 56, as $e f$ will always be less than $d g$. In practice, however, the difference between these angles is not appreciable, and no sensible error will result from considering them as identical.

201. The whole of the projections, bevels, and templets, above described, are shown in a connected form in fig. 77, plate 7; a careful study of which will materially assist the reader in obtaining a clear understanding of the principles which we have endeavoured to explain. There is, however, so much difficulty in understanding the nature of spiral planes without models, that we would recommend the reader to procure a wooden cylinder, say, three ft. diameter, and to work out upon it all the problems connected with skew masonry. The deve-

* That is, when the rules are applied to the length of the stone.

lopments may be made on drawing-paper, and their accuracy tested by bending them round the model. The templets and bevels may be cut out of card-board; and the accuracy of the face bevels may be proved by setting up in card-board an elevation of the face, and trying them against it. The construction of a model of this kind is the best method of obtaining a knowledge of the subject, and more will be learnt by this means in a few days than could ever be done by the study of drawings alone.

GROINED VAULTING.

202. *Roman Vaulting*.—The principles of Roman vaulting have been explained at considerable length in section I.; and in section II., articles 119 to 131, the methods of obtaining the profiles of the groins, and the developments of the soffits of cylindrical vaults, have been fully shown. We have, therefore, in this place only to describe the application of these principles to the working of the groins, no other portion of a common groined vault offering any particular difficulty.

203. The simplest way of working a groin-stone is to bring the stone into a cubical form, as shown in fig. 92, plate 8; and on the vertical and horizontal surfaces of operation thus obtained, to apply templets taken from a full-sized plan and elevation, see fig. 91, plate 8. This is the easiest way of proceeding; but the waste of stone is very considerable.

204. If the stone to be worked is only sufficiently large to contain its intended form without any waste, we must begin by working two plane faces at right angles to each other, to contain the heading joints *a b c d*, *b i g h*, fig. 92, plate 8. These having been worked, and the form of the stone marked with a templet taken from a

RUDIMENTS 'OF THE

full-sized section of the vault, the top and bottom beds can be worked with a common square, and the arris lines drawn upon them. The curved soffits can then be finished with a curved rule, cut to the proper curve and applied between the top and bottom arrises. This method makes the most of the stone and saves the labour of making surfaces of operation; but it requires considerable care to keep the angles perfectly true.

205. The above methods suppose that the main and the cross vault are built in horizontal courses, which would always be done in the Italian style; but it is quite possible to keep the courses of the main vault horizontal, and to make those of the cross vault radiate from the centre of the main vault. This arrangement may be seen in fig. 6 (art. 17). It is only suited to rough rubble work, as the execution of such a vault in regular masonry would be a most complicated process.

206. *Gothic Vaulting*.—In Gothic vaulting, as explained in section I., the profiles of the groins are always formed of circular curves, and the forms of the vaulting surfaces are made to depend on the curvature of the groins, instead of the groins following the form of the vaulting surface, as in Roman vaulting.

It is true that, in the decline of the pointed style, elliptical groins were used to a certain extent; but this was after the introduction of vaults of solid masonry, as the fan vault, and the later lierne vaults, which assimilate very closely in their construction, although not in their decoration, to the vaults of the modern Italian school.

For this reason we do not propose here to take into consideration the construction either of fan vaults or of the late pointed vaults, which are chiefly built of jointed masonry.

The construction of a pointed waggon vault of solid

masonry is precisely similar in principle to that of a common cylindrical arch, however complicated the tracery which may be sunk upon its soffit; and the construction of a fan vault may be accomplished either by the rules given in art. 158 and following articles, or by forming horizontal surfaces of operation, as shown in fig. 10, art. 25; which seems to have been the plan adopted by the masons of the middle ages; although in many existing vaults the extrados is parallel to the soffit, the surfaces of operation having been chipped off, so as to bring the upper surface of the vault to a curved form.

207. Rib and Pannel vaulting is quite different in its construction, both from Roman vaulting and from the late pointed vaults of which we have just spoken.

It consists, as has been before explained, of a frame-work of light ribs, each of which is worked in the same manner as a cylindrical arch; and of light pannels which rest on this frame-work, and are either built in courses or formed of thin slabs of stone scribed to the ribs; the general principle on which the vaulting surfaces are formed being, that the soffits of the pannels should everywhere coincide with a straight-edge applied in a horizontal direction from rib to rib; although when the pannels are built in courses they are made slightly concave as the stones would otherwise have little to keep them from falling. No difficulty occurs in working the ribs themselves, since each stone forms a portion of a cylindrical arch; but a considerable amount of projection and transference of lines is required in arranging the curves of the ribs, and to obtain the bevels for working the stumps of the ribs on the boss-stones, at their inter-sections. We propose, therefore, to conclude this little volume by a brief description of the projections required for the execution of a plain ribbed vault, with an expla-

nation of the manner of finding the curvature of the liernes and the bevels for the boss-stones in the simplest class of lierne vaults.

208. The various ribs introduced in Gothic vaulting may be classed under six heads, viz. :—

1st. Transverse ribs, which are placed at right angles to the length of the vault.

2nd. Longitudinal ribs, which are parallel to the length of the vault. If the apartment be vaulted in one span, the longitudinal ribs are called, from their position, *wall ribs*.

3rd. Diagonal ribs, or cross springers. Upon these the main strength of a Gothic vault depends; whilst, in the Roman groined vault, without ribs, the groins are the weakest parts.

4th. Intermediate ribs. These are ribs introduced between the transverse and diagonal ribs, and may be either surface ribs, that is, ribs coinciding with a previously determined vaulting surface; or they may be independent ribs, each of which marks a groin; that is, a change in the direction of the vaulting surface.

5th. Ridge ribs. Ridge ribs, as essential portions of the construction of a vault, are unnecessary where no intermediate ribs are introduced; and, in this case, the ridge ribs of the Gothic vaults were frequently built in with the pannels instead of being previously built as a portion of the framework of the vault. An example of this may be seen in a vault in the ruins of Wingfield Manor House, Derbyshire. In this vault, the central bosses have been prepared for the reception of the ridge ribs; but the latter, instead of being moulded to correspond with the mouldings of the bosses, are plain canted strips, built in as keystones to the rubble arches forming the pannels. Where intermediate ribs are introduced,

the ridge ribs become essential as struts to keep the former in their place previous to the insertion of the pannels.

In making the ridge ribs form part of the framework of the vault to be built with a light skeleton centre, a difficulty occurs, unless the vault be highly domical in its structure; as there is otherwise nothing but the centering to keep them in their places until they can be supported by the pannels. A common remedy for this was to make each portion of the ridge from boss to boss slightly concave. A very striking example of this may be seen at Lincoln Cathedral; where the bosses appear to be placed in a level line, or nearly so, whilst the ridge ribs of the several compartments form a series of flat arches between them.

6th. *Liernes*. These are short ribs introduced between the principal ribs, so as to form ornamental patterns. Their forms are generally governed by the vaulting surface, although they are built as separate arches, not as portions of the pannel. When many liernes are introduced, the construction of the vault becomes complicated; and, instead of the skeleton centre, which is all that is requisite for constructing a plain ribbed vault, a regular boarded centering must be provided. In the complex lierne vaults, the principle of the plain ribbed vault, viz., the making the vaulting surface to depend upon the curvature of the ribs, is, in a great measure, lost sight of; as it becomes necessary first to design the general form of the vault, with which the curves of the ribs must be made to correspond.

209. *Curvature of the Ribs*.—In designing a plain ribbed vault, it is simplest to begin with the transverse ribs, as their form, in a great measure, governs the appearance of the work.

Each rib may be struck as a single arc of a circle, or from two centres; so that each pair of ribs forms a four-centered arch. Whichever plan is adopted, the centre of the curve at the springing should be *on* the springing line; neither above nor below it, as either of these positions produces an unpleasant effect; the curve in the former case becoming horse-shoed, and, in the latter, forming an acute angle with the springing line.

Fig. 93, plate 8, shows the plan of a quarter of one compartment of a ribbed vault, with the elevation of each rib placed on its plan—*a l b* is the plan, and *a a' B*, the elevation, of the transverse rib.

210. The transverse ribs having been decided on, the next thing to be settled is the form of the cross-springers, and here some little arrangement is necessary. Two objects should be kept in view; the first, to make the radius of curvature as nearly as possible the same as that of the transverse ribs; and the second, to make the curve at the springing start at right angles to the springing line. The simplest way of accomplishing these objects is to strike the transverse and the diagonal ribs with the same radius; the centre of the curve being placed in both cases on the springing line. This is shown in fig. 94, plate 8, where *a B* is the elevation of the transverse, and *a c* that of the diagonal, rib. This was a common arrangement in continental vaulting; but it has the peculiarity of producing a highly-domed vault, the intersection of the cross-springers at *c* being much above that of the transverse ribs at *B*. If we wish to keep the ridges horizontal, we have a new condition introduced, and the complete solution of the problem cannot be effected with single arc ribs only. If we confine ourselves to ribs formed of a single arc, we may make the diagonal rib of the same radius as the trans-

verse; placing the centre below the springing line, as in fig. 95, plate 8; or we may keep the centre of the diagonal rib on the springing level, and diminish the radius, as shown in fig. 96, plate 8.

By the employment of two-centered ribs, however, the adjustment of the curvature can be accomplished with great facility; this is shown in fig. 97, plate 8, where the first portions of the diagonal and transverse ribs are struck with a common radius ad ; the remainder of the transverse rib being struck from f , and that of the diagonal rib from e , so that each pair of diagonal ribs forms a three-centered arch, of which the flatness at the crown is concealed by the boss. These examples will suffice to show the variety of ways in which the curvature of the diagonal ribs may be determined.

211. The curvature of the cross-springers determines the general plan of the spandril, and this governs, to a certain extent, the curvature of the intermediate ribs. The longitudinal rib may be determined either as shown in figures 96 and 97, or the proportion of the rise to the span may be kept the same as in the transverse ribs and the springings stilted, as shown in the elevation of the wall rib $ee'f$, fig. 93. This last arrangement was a very common one in church roofs; the stilting of the wall ribs being necessary in order to leave proper space for the clerestory windows.

To ascertain the general plan of the spandril solid, take any point as a^1 half-way up the transverse rib, and let fall the perpendicular $a^1 1$. Set off this height on the elevations of the diagonal and wall ribs, and from the points c^1, e^1 where a horizontal line at this level cuts the ribs, let fall the perpendiculars $c^1 2$ and $e^1 3$. Join $1, 2$; $2, 3$, then $a, 1\ 2\ 3\ e$ is the general plan of the spandril at the height a^1 .

We have already spoken (section I., art. 19) of the variety of form that may be produced in the middle plan of the spandril by a slight alteration in the curvature of the ribs; and the reader will, therefore, understand, without further explanation, the method about to be described of obtaining the curves of the intermediate ribs from this middle plan. Thus let it be determined to make the intermediate ribs project before the lines 1, 2; 2, 3. Design the plan $a\ 1\ 4\ 2\ 5\ 3\ e$, so as to give to the spandril the form that may be wished; and from the points 4, 5, erect the perpendiculars $4\ g^1$, $5\ l^1$, each respectively equal to $1\ a^1$. Then the form of the ridge having been previously decided on, (in the example shown in fig. 93, both the longitudinal and transverse ridges are made horizontal,) we have three points in each intermediate rib through which to draw the required curve, which may be struck either from one or two centres, according to circumstances. In fig. 93, the whole of the ribs are made single arcs of circles; the centre of the intermediate rib ik being placed above, and that of the rib gh being placed below, the springing level. If the ridge-ribs are not horizontal, their elevations must be drawn before those of the intermediate ribs, and the points h , k , ascertained accordingly.

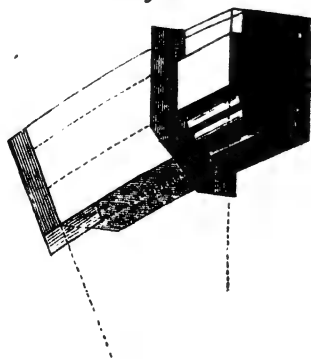
212. For the purpose of showing the manner of finding the curvature of a lierne lying in a given vaulting surface, we have introduced in fig. 93 two liernes ih and ik . From any convenient point m on the diagonal rib, about half-way between i and d , set up the perpendicular $m\ m^1$, and from i set up the perpendicular $i\ l$. Draw $m\ m^1$, $i\ l^1$, parallel to the springing. On the elevation of the intermediate rib gh set off $h\ n^1 = d\ m^1$; draw $n^1\ n$ parallel to the springing, and from n let fall the perpendicular $n\ n$. Join $n\ m$, cutting ih at p ; $n\ m$

will be the plan of a level line on the soffit. On ik erect the perpendiculars hH^1 and pP , making $hH^1 = nL^1$ and $pP = m^1L^1$. Then i, P and H^1 are three points in the soffit of the lierne, through which the required curve may be readily drawn with a trammel.

The curvature of the lierne ik is obtained in the same manner.

213. The voussoirs forming the ribs are worked in a very simple manner, as each stone forms a portion of a cylindrical arch. Two parallel faces of operation are first formed, at a distance apart equal to the maximum thickness of the rib, and on one of these faces the curve of the soffit is marked with a templet. The soffit is then worked with a common square, and the ends of the stone cut to radiate from the curve of the soffit, either with an arch-square or with a templet applied on one of the faces. The profile of the mouldings is then marked on the ends with a templet, and the soffit

Fig. 98.



gauged to its proper width; lastly, the lines bounding the parallel faces of the rib are scribed on the latter with a gauge applied to the soffit, and the mouldings are sunk by means of a mould applied to these lines and to the arris lines of the soffit, as shown in fig. 98, which, however, is only intended to show the

principle of operation, as practically each member is worked in succession, with a separate templet, surfaces of operation being formed, containing the arrises of the mouldings between which the templets are applied.

A rebate must be sunk in the upper part of each face to receive the pannels.

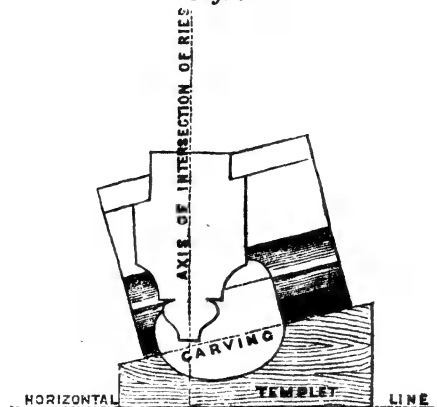
214. In arranging the positions of the feet of the ribs upon the impost, care must be taken to make each rib as much as possible distinct and independent; which is done by making the ribs start at different distances from the intersections of their centre lines. This will be understood by reference to fig. 93; in which the intermediate ribs are made to spring from behind the intersections of the diagonal and transverse and longitudinal ribs. The ribs cease to be worked as separate arches, from the level at which the mouldings begin to intersect each other. Below this point, the springing must be worked in horizontal courses; the upper bed of the top course being cut into a series of inclined planes, so as to form a proper abutment for the foot of each rib.

To work these springers the top and bottom beds are first worked, and the centre lines of the ribs marked upon them. The position of the soffit of each rib is then transferred to these lines from a full-sized elevation, and the soffits worked with **convex** templets applied to the top and bottom arrises. Lastly, the soffits are gauged to the proper widths, and the mouldings worked out with moulds applied in a direction radiating from the curve of each rib.

215. In working the key-stones at the intersections of the ribs some little difficulty occurs, inasmuch as from each rib being a separate arch, its middle section must be a vertical plane, and the mouldings of the ribs will, therefore, not mitre, but intersect each other in a very awkward manner; see fig. 99.

To hide this, the key-stones are worked with a round lump or *boss*, ornamented with foliage and sculpture; so

Fig. 99.

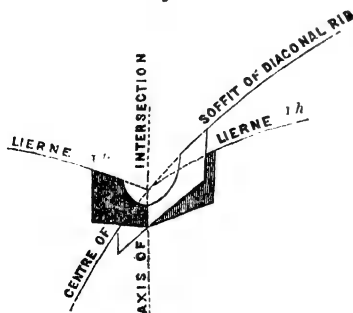


a large block, and to work upon it an upper horizontal surface of operation, to which the centre lines of the ribs were transferred from a full-sized plan of the vault; and the form of the soffit was then obtained by squaring down from this upper surface.

217. This method occasions much extra labour, and a great waste of stone. The following is preferable:— Suppose it is required to work the boss-stone at 1, fig. 93, plate 8.

Make a short templet to the curve of the soffit of the

Fig. 100.



rib *cd*, so that its bottom edge shall be horizontal; and mark on it a vertical line, corresponding to the axis of intersection of the ribs (see fig. 100). Make similar templates for the two liernes, *1 h*, *1 k*. Mark on each

templet the form of the boss, and cut away the upper edges; so that when the three templets are applied to the soffit of the finished stone, the ends of each shall exactly coincide with the soffit of each rib; the carving of the boss lying in the hollows thus formed. To work the stone, begin by sinking a draft to the templet for the diagonal rib; and then, placing one of the lierne templets at the proper angle with the first templet, sink it into the stone until the *horizontal* edges of the two are out of winding. Apply the third templet in a similar manner. Knock off the superfluous stone square to the drafts, and we have then three curved surfaces of operation containing the soffits of the three ribs. The rest of the operation presents no difficulty.

The above method applies to all the boss-stones in a vault, whatever their shape or position, and will be understood by inspection of figs. 99 and 100.

218. The stumps on the boss-stones should always be squared to form proper abutments for the ends of the ribs. This was not always done by the mediæval architects, who often worked the stumps of the liernes on the bosses, so as to form acute angles with their soffits.

